Composite Figures

Lesson Synopsis:

In this lesson students design rectangular regions and collect data to investigate relationships involving perimeter and area. Students use multiple representations to explore perimeter, circumference, and area and make connections between geometry and algebra. Students use basic area formulas in a new setting as they solve perimeter, circumference, and area problems involving composite figures, and explore the effects of dimensional change on perimeter, circumference, and area.

TEKS:

G.4	***Geometric structure. The student uses a variety of representations to describe geometric relationships and solve problems.***
G.4	**Select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.**
G.5	***Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems.***
G.5A	**Use numeric and geometric patterns to develop algebraic expressions representing geometric properties.**
G.5B	**Use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ~~ratios in similar figures and solids, and angle relationships in polygons and circles.~~**
G.8	***Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.***
G.8A	**Find areas of regular polygons, circles, and composite figures.**
G.8B	**Find areas of sectors and arc lengths of circles using proportional reasoning.**
G.8E	**Use area models to connect geometry to probability and statistics.**
G.8F	**Use conversions between measurement systems to solve problems in real world situations.**
G.9	***Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures.***
G.9B	**Formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models.**
G.9C	**Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.**
G.11	***Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems.***
G.11D	**Describe the effect on perimeter, and area~~, and volume~~ when one or more dimensions of a figure are changed and apply this idea in solving problems.**

GETTING READY FOR INSTRUCTION

Performance Indicator(s):

- Find the perimeter and area of polygons and composite figures. Convert between measurement systems as necessary to solve problems. Describe the effects changing one or more dimensions of the figure have on perimeter and area. (G.4; G.5A, G.5B; G.8A, G.8B, G.8F; G.9B, G.9C; G.11D)
  ELPS ELPS: 1E, 1H, 2E, 2I, 3H, 4F, 5G
- Use area models to determine and represent probability. Connect probability to geometry by finding and comparing areas of geometric figures. (G.4; G.8A, G.8B, G.8E)
  ELPS ELPS: 1E, 1H, 2E, 2I, 3H, 4F, 5G

Key Understandings and Guiding Questions:

- Perimeter and area of polygons and composite figures can be found by various methods, including formulas.
  — What methods can be used to find the perimeter of polygons and regular polygons?
  — What methods can be used to find the area of polygons and regular polygons?
  — What is a composite figure?

- What methods can be used to find the perimeter and area of composite figures?
- Units may be converted from one measurement system to another in a problem situation.
  - Who uses the customary measurements?
  - Who uses metric measurements?
  - Why might it be important to be able to convert from one measurement system to the other?
  - How can the conversion number 1 inch equals 2.54 centimeters be used to convert from inches to centimeters or vice versa?
- Changing one or more of the dimensions of a figure will have a predictable effect on the perimeter and area of the new figure.
  - How does changing one dimension of a figure affect its perimeter?
  - How does changing one dimension of a figure affect its area?
  - How does changing two dimensions of a figure affect its perimeter?
  - How does changing two dimensions of a figure affect its area?
- Geometric figures can be used to create area models to represent probability.
  - How can a geometric figure be used as an area model to represent a given probability?

Vocabulary of Instruction:

- perimeter
- circumference
- area
- composite figures
- dimensional change

Materials:

- graphing calculator
- cm ruler
- a loop of string
- thumbtacks
- bulletin board
- colored pencils
- card stock (optional)
- safety compass (optional)
- ruler (optional)
- beans, cereal, or candy (optional)
- circular objects (optional)
- ribbon (optional)
- scissors (optional)
- chart paper and markers (optional)
- meter sticks (optional)
- tape or glue stick (optional)
- sticky dots (optional)

Resources:

- **STATE RESOURCES**
  - **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments http://www.utdanacenter.org/mathtoolkit/index.php
  - **TEXTEAMS: High School Geometry: Supporting TEKS and TAKS:** IV – Planar Figures; 3.0 Bayou City Lake, 3.1, Act. 1 (Bayou City Lake)

Advance Preparation:

1. Handout: **Nelly's on the Run!** (1 per student)
2. Handout: **Nelly's Got Company!** (1 per student)
3. Handout (optional): **Ring Around the Roses** (1 per student)
4. Handout (optional): **Going Around in Circles Teacher Notes** (1 per teacher)
5. Handout (optional): **Going Around in Circles** (1 per student)
6. Handout: **Perimeter, Circumference, and Area** (1 per student)
7. Handout: **Measuring Composite Figures** (1 per student)
8. Handout: **Area Models and Probability** (1 per student)
9. Handout: **Poolside** (1 per student)
10. Handout (optional): **Sky Man** (1 per student)
11. Handout: **Evaluating Composite Figures** (1 per student)

Background Information:

The focus of this lesson is on perimeter and area relationships that result from dimensional change and from composite figures. As such, students must have a working knowledge of area formulas from middle school mathematics in order to be prepared for the explorations. This lesson is a precursor to dimensional change of solids in the upcoming unit as students will explore the effects of dimensional change on surface area and volume.

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, and substitute resources, materials, and activities to differentiate instruction to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource in your district Curriculum Developer site for future reference.

INSTRUCTIONAL PROCEDURES

Instructional Procedures

ENGAGE

1. Distribute copies of handout: **Nelly's on the Run!** to each student.
2. Have students work in groups to complete p. 1.
3. Debrief students in whole group discussion. If necessary, students may complete the handout for homework and debrief at the beginning of the next period.
   Facilitation Questions:
   - **How many different rectangular goat pens of whole number side lengths were you able to sketch?** *Eleven.*
   - **Are the perimeters of all the figures the same?** *Yes, all have a perimeter of 24 units.*
   - **Are the areas of all figures the same?** *The areas of the figures are different.*
   - **Based on your sketches alone, are you able to find the area of each pen? Explain.** *Yes. Since each grid block is 1 yd^2, the area of each pen can be found by counting the number of grid blocks enclosed by the pen.*
   - **Based on your sketches, which pen has the greatest area?** *The 6 yard by 6 yard square pen.*

Notes for Teacher

NOTE: 1 Day = 50 minutes
Suggested Day 1 (1/4 day)
MATERIALS
- Handout: **Nelly's on the Run!** (1 per student)
- Handout: **Nelly's Got Company!** (1 per student)
- graphing calculator
- colored pencils

TEACHER NOTE
The purpose of the ENGAGE is to get students to start thinking about area and perimeter relationships.

STATE RESOURCES
Mathematics TEKS Toolkit: Clarifying Activity/Lesson/Assessment may be used to reinforce these concepts or used as alternate activities.

EXPLORE/EXPLAIN 1

1. Have students work in groups to complete p. 2-4.
2. Debrief students in whole group discussion. If necessary, students may complete the handout for homework and debrief at the beginning of the next period.
   Facilitation Questions:
   - **Based on the data you collected, is there a relationship between the width of the rectangles and the length of the rectangles as the perimeter is held constant at 24 yards? Explain.** *Yes. The width decreases by 1 yard as the length increases by 1 yard each time.*
   - **What does this relationship look like graphically? Algebraically?** *Graphically, the relationship is linear; algebraically, $W = 12 – L$.*
   - **Based on the data you collected, is there a relationship between the area of the rectangular pens and the length of the pens? Explain.** *Yes. Algebraically, $A = 12L – L^2$.*
   - **What does this relationship look like graphically?** *It is a parabola.*
   - **Based on the area function $A = 12L – L^2$, what is the maximum area and how is it reflected on the graph of the function?** *The function takes on a maximum at a length of 6 yards, which makes the width also 6 yards; hence, the maximum area occurs with the 6 yard by 6 yard square pen. On the graph, it is the vertex of the parabola.*
3. Distribute copies of handout: **Nelly's Got Company!** to each student.
4. Have students continue to work in their group to complete the activity.
5. If necessary students may complete the activity as homework.

Suggested Day 1 (3/4 day)
MATERIALS
- Handout: **Nelly's Got Company!** (1 per student)
- graphing calculator

TEACHER NOTE
The purpose of this activity is to explore how area of a figure changes as the perimeter is held constant. In the activity, the students design rectangular goat pens of a given perimeter and analyze the relationship concretely: from a table, graphically, and algebraically.

TEACHER NOTE
Make sure to explore the connections between the various representations of the problem situation in the previous activity. Student understanding will vary, and the teacher should be prepared to ask additional facilitation questions to help clarify student understanding.

TEACHER NOTE
The purpose of **Nelly's Got Company**

Instructional Procedures	Notes for Teacher
	is to investigate the effects of dimensional change on perimeter and area of a figure. From the previous activity, students discovered that the 6 yard by 6 yard pen yields the greatest area. In this activity, students investigate what happens to the perimeter and area of the 6 yard by 6 yard pen as one side or both are lengthened. **SUPPLEMENTAL MATERIALS** • Handout (optional): **Ring Around the Roses** (1 per student) • Handout (optional): **Going Around in Circles Teacher Notes** (1 per teacher) • Handout (optional): **Going Around in Circles** (1 per student) • card stock • safety compass • ruler • beans, cereal, or candy • circular objects • ribbon • scissors • chart paper and markers • meter sticks • tape or glue stick • sticky dots These activities look at the mathematical relationship between the radius, diameter, circumference, and area of circles. If a day is available, one of these activities can be used as an additional activity. No keys are provided for these activities, since answers will vary according to data collected.
EXPLORE/EXPLAIN 2	**Suggested Day 2**
1. If not done in the previous period, debrief the activity, **Nelly's on the Run!** using the facilitation questions. Facilitation Questions: • **According to your sketches, how does lengthening one side of the square pen change the perimeter of the pen and the area of the pen? Explain.** *Lengthening the pen along one side by one yard increases the perimeter by 2 yards each time and the area by 6 yds^2 each time.* • **What does this lead you to believe about the relationship between lengthening the square along one side and the perimeter, and lengthening the square along one side and the area?** *The relationship between lengthening the square along one side and the perimeter is linear. The relationship between lengthening the square along one side and the area is linear.* • **How can these relationships be represented algebraically and how do they connect to your sketches?** *The perimeter formula becomes $P = 2(6 + x + 6)$ where $(6 + x)$ is the side length that is*	**MATERIALS** • Handout: **Perimeter, Circumference, and Area** (1 per student) • graphing calculator **TEACHER NOTE** The purpose of this activity is to review the formulas for perimeter, circumference, and area of polygons and circles and to apply these to solve problems. **TEACHER NOTE** Be sure to connect the area models from the student sketches to the algebraic representations to enhance

Instructional Procedures

increasing and 6 is the side length that is constant. The area formula becomes A = (6 + x)6, where (6 + x) is the side length that is increasing and 6 is the side length that is constant.

- **According to the sketches, how does lengthening both sides of the square pen change the perimeter of the pen and the area of the pen? Explain.** *Lengthening the pen along both sides by one yard increases the perimeter by 4 yards each time. Lengthening the pen along both sides by one yard increases the area by 13 yards, then 15 yards, then 17 yards, and so on.*
- **What does this lead you to believe about the relationship between lengthening the square along both sides and the perimeter, and lengthening the square along both sides and the area? Explain.** *Since the perimeter increases by 4 yards each time, the relationship is linear. Since the area does not increase by the same amount each time, the relationship is not linear.*
- **How can these relationships be represented algebraically and how do they connect to your sketches?** *The perimeter formula becomes P = 2(6 + x + 6 + x) where (6 + x) represents both the length and width of the square or rectangle. The area formula becomes A = (6 + x)(6 + x) where (6 + x) represents both the length and width of the square or rectangle.*

2. Distribute the handout: **Perimeter, Circumference, and Area** to each student.
3. Go over the notes on p. 1 in whole group instruction as students fill in their handout. Direct students to pay close attention to the piece of the TAKS Mathematics Formula Chart and how the formulas are presented on the chart.
4. Have students work the Practice Problems in pairs. Students can complete the handout as homework, if necessary.

Notes for Teacher

understanding. The sketches model area concretely as the product of length and width much like algebra tiles are used to model multiplication of a binomial and a monomial, and a binomial and a binomial. This should be pointed out to enhance student understanding of why area changes as a quadratic when both length and width are changed.

EXPLORE/EXPLAIN 3

1. Distribute the handout: **Measuring Composite Figures** to each student.
2. Go over the paragraph at the top of p. 1 and work problem #1a,b,c in whole group instruction as students fill in their handout. Students will need a centimeter ruler to complete #1a.
3. Have students work the remaining problems #2-5 in pairs. Students can complete the handout as homework, if necessary.

Suggested Day 3

MATERIALS

- Handout: **Measuring Composite Figures** (1 per student)
- ruler (centimeter)
- graphing calculator

TEACHER NOTE

The purpose of this activity is to investigate methods to determine perimeter, circumference, and area of composite figures and to apply these to solve problems.

EXPLORE/EXPLAIN 4

1. Distribute the handout: **Area Models and Probability** to each student.
2. Go over the notes at the top of p. 1 in whole group discussion.
3. Have students work in pairs to complete p. 1.
4. Ask volunteers to share answers in whole group.
5. Work problems #6 and 9 as samples.
6. Have students continue to work in pairs on the remainder of the problems. If necessary, the activity may be completed as homework.

Suggested Day 4

MATERIALS

- Handout: **Area Models and Probability** (1 per student)
- graphing calculator

TEACHER NOTE

The purpose of this activity is to investigate methods to use area models to both represent and determine

Instructional Procedures

Notes for Teacher

probability.

ELABORATE

1. Distribute the handout: **Poolside** to each student.
2. Have students work the activity in pairs. Students can complete the handout as homework, if necessary.

Suggested Day 5

MATERIALS

- Handout: **Poolside** (1 per student)
- graphing calculator

TEACHER NOTE

The purpose of this activity is to determine perimeter, circumference, and area of composite figures in problem situations and to predict the effects of changing dimensions on perimeter and area.

SUPPLEMENTAL MATERIALS

- Handout (optional): **Sky Man** (1 per student)
- ruler

This activity incorporates finding area of composite figures and probability. No key is available for this activity, since measurements will vary depending on copier machines.

STATE RESOURCES

TEXTEAMS: High School Geometry: Supporting TEKS and TAKS: IV – Planar Figures; 3.0 Bayou City Lake, 3.1, Act. 1 (Bayou City Lake) may be used to reinforce these concepts or used as alternate activities.

EVALUATE

1. Distribute the handout: **Evaluating Composite Figures** to each student.
2. Have students complete the handout independently as an assessment of perimeter, circumference, area, and composite figures.

Suggested Day 6

MATERIALS

- Handout: **Evaluating Composite Figures** (1 per student)
- graphing calculator

TEACHER NOTE

The purpose of this activity is to assess student understanding of the concepts covered in this lesson.

TAKS CONNECTION

Grade 9 TAKS 2003 #16,21,26,31,34
Grade 10 TAKS 2003 #8,12,32,38,55
Grade 11 TAKS 2003 #9,51,52

Grade 9 TAKS 2004 #29,35,37,49
Grade 10 TAKS 2004 #5,15,19,21,50
Grade 11 TAKS 2004 #7,26,43,46,52
Grade 11 July TAKS 2004

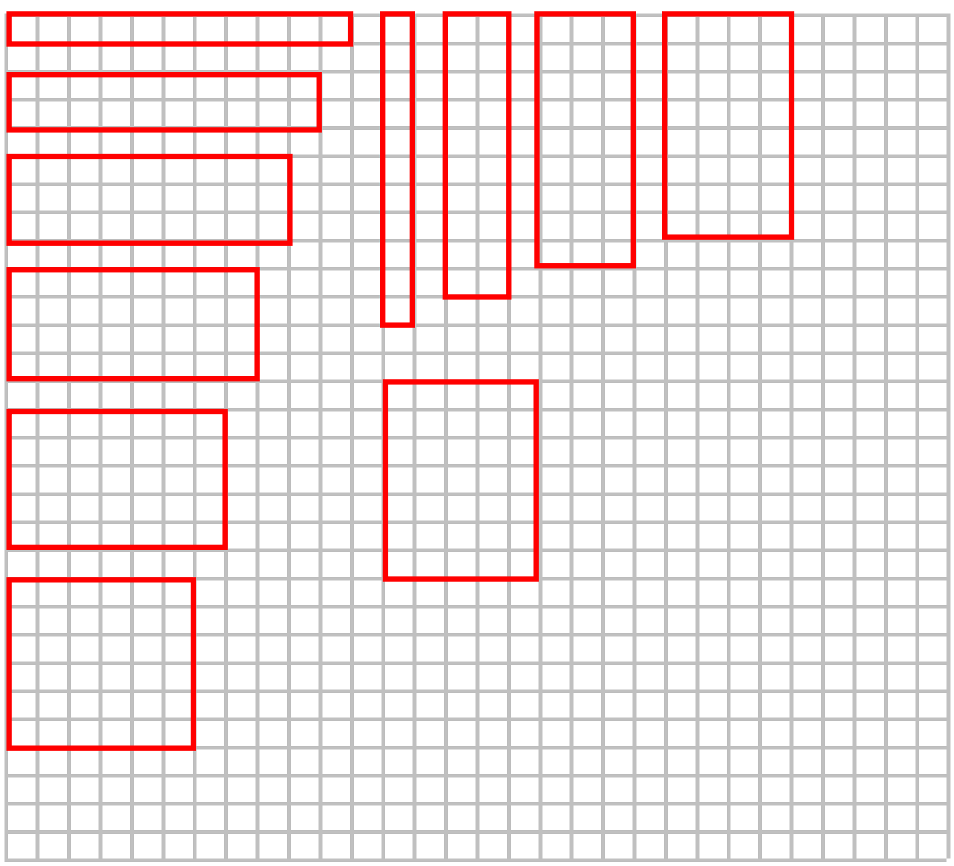
Instructional Procedures	Notes for Teacher
	#4,7,11,24,46 Grade 9 TAKS 2006 #7,9,18,26 Grade 10 TAKS 2006 #23,25,43 Grade 11 TAKS 2006 #22,33,35,47 Grade 11 July TAKS 2006 #6,18,45

Nelly's on the Run! (pp. 1 of 4) KEY

Your prized Nubian Goat, Nelly, is at it again! Just the other day you tethered her to the barn using a piece of rope because she was caught grazing in your neighbor's pepper patch. It seems as if she has chewed through the rope and is on the run. You decide to build a temporary pen using some left over fencing from another project. The left over fencing is 24 yards long and you begin to wonder what size rectangular pen will allow Nelly the most area to graze.

Holding Perimeter Constant

1. Use the grid below to sketch all rectangular pens of whole number side lengths that could be built using the entire 24 yards of fencing. Be sure to sketch pens that have the same dimensions but different corresponding lengths and widths. Ex: 1 x 11 and 11 x 1 pen. (Do not allow for a gate or other opening.) Assume each block on the grid is 1 yard by 1 yard.



Nelly's on the Run! (pp. 2 of 4) KEY

2. Based on your sketches, which pen appears to enclose the most area?
   The 6 yd by 6 yd pen appears to enclose the most area.

3. Based on your sketches, how could you find the area of the pens without calculating them? Explain your reasoning.
   Since each grid block represents 1 yd by 1 yd, or 1 yd^2, and the side lengths are whole numbers, the area of each pen can be found by counting the number of blocks enclosed by each rectangle.

4. Based on your answer to question 3, find the areas of the rectangular pens in the sketch.
   See student sketches on grid paper.

5. Use your sketch of the rectangular pens to complete the data table below. Be sure to list pens that have the same dimensions but different corresponding lengths and widths. For example, include a 1 yd by 11 yd pen as well as an 11 yd by 1 yd pen.

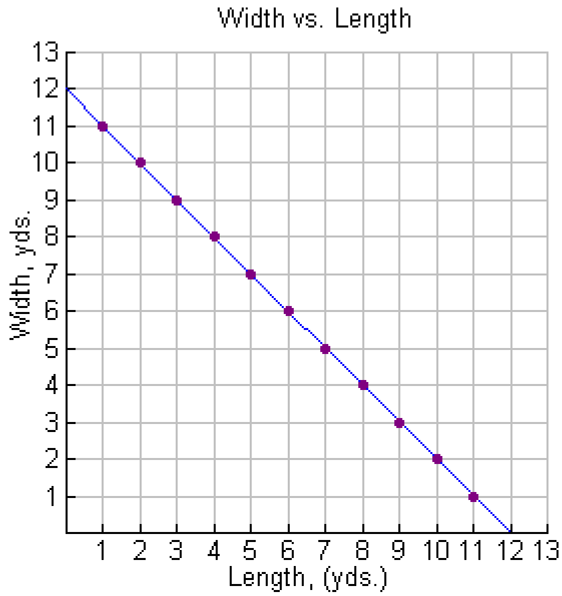
Length (yd)	Width (yd)	Process for Perimeter	Perimeter (yd)	Process for Area	Area (yd^2)
1	11	2(1 + 11)	24	(1)(11)	11
2	10	2(2 + 10)	24	(2)(10)	20
3	9	2(3 + 9)	24	(3)(9)	27
4	8	2(4 + 8)	24	(4)(8)	32
5	7	2(5 + 7)	24	(5)(7)	35
6	6	2(6 + 6)	24	(6)(6)	36
7	5	2(7 + 5)	24	(7)(5)	35
8	4	2(8 + 4)	24	(8)(4)	32
9	3	2(9 + 3)	24	(9)(3)	27
10	2	2(10 + 2)	24	(10)(2)	20
11	1	2(11 + 1)	24	(11)(1)	11

   a. According to the data in your table, describe how the width of the rectangle changes as the length increases.
      The width decreases by 1 yard as the length increases by 1 yard each time.

   b. What does your answer to 5.a lead you to believe about the relationship between the width and length of a rectangle of perimeter 24?
      The relationship is linear with negative correlation.

Nelly's on the Run! (pp. 3 of 4) KEY

c. Using your graphing calculator and the data in your table, create a scatterplot of width vs. length for the rectangle of perimeter 24 yards. Sketch the scatterplot below.



d. Based on the data in your table and the scatterplot, write an equation for the width of the rectangle in terms of its length.
$W = 12 - L$

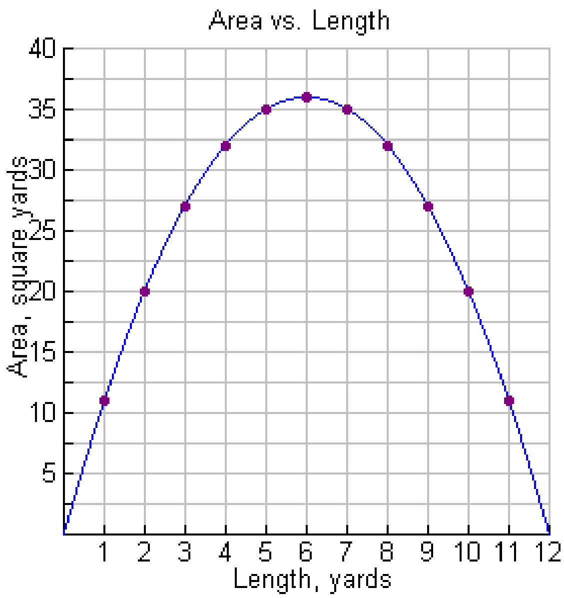
e. Enter the equation from 5.d as a function in the graphing calculator. Graph the function along with the scatterplot. Sketch the graph along with your scatterplot in 5.c.
See 5.c.

f. Given the context of the problem, what is an appropriate domain and range for the function that relates width to length? Explain your reasoning in terms of the problem situation.
The length cannot be zero since that would not result in a rectangle. Since the perimeter is always 24 yards, a length of 12 yards would result in zero width which is not possible. Therefore the Domain = $\{0 < L < 12\}$. The Range = $\{0 < W < 12\}$ by similar reasoning.

g. Suppose you wanted to use your function to construct rectangles of perimeter 24 yards with non-whole number side lengths. Find the width given a length of 7.25 yards. Find the length given a width of 6.6 yards. Write your answers as ordered pairs that satisfy your equation from 5.d.
(7.25, 4.75) and (5.4, 6.6)

Nelly's on the Run! (pp. 4 of 4) KEY

6. Using your graphing calculator and the data in the table, create a scatterplot of area vs. length for the rectangle of perimeter 24 yards. Sketch the scatterplot below.



a. Based on the data in your table, write an equation for the area in terms of the length of the rectangle with perimeter 24.
$A = 12L - L^2$

b. Enter the equation from 6.a as a function in the graphing calculator. Graph the function along with the scatterplot. Sketch the graph along with your scatterplot in question 6.
See question 6.

c. Given the context of the problem, what is an appropriate domain and range for the function that relates area to length? Explain your reasoning in terms of the problem situation.
The length cannot be zero since that would not result in a rectangle. Since the perimeter is always 24 yards, a length of 12 yards would result in zero width which is not possible. Therefore the Domain = $\{0 < L < 12\}$. The area takes on positive values greater than zero until the area is reaches is maximum, 36 yds.^{2}; therefore, the Range = $\{0 < A \leq 36\}$.

d. Based on the area function, which rectangular pen will result in the most area? Is this the same or different from your answer in question 2? Explain.
The area function takes on a maximum value for a length value of 6. The answers are the same. Since the perimeter is 24, the width is 6 when the length is 6, therefore, the rectangle of maximum area is actually a square.

Nelly's on the Run! (pp. 1 of 4)

Your prized Nubian Goat, Nelly, is at it again! Just the other day you tethered her to the barn using a piece of rope because she was caught grazing in your neighbor's pepper patch. It seems as if she has chewed through the rope and is on the run. You decide to build a temporary pen using some left over fencing from another project. The left over fencing is 24 yards long and you begin to wonder what size rectangular pen will allow Nelly the most area to graze.

Holding Perimeter Constant

1. Use the grid below to sketch all rectangular pens of whole number side lengths that could be built using the entire 24 yards of fencing. Be sure to sketch pens that have the same dimensions but different corresponding lengths and widths. Ex: 1 x 11 and 11 x 1 pen. (Do not allow for a gate or other opening.) Assume each block on the grid is 1 yard by 1 yard.

Nelly's on the Run! (pp. 2 of 4)

2. Based on your sketches, which pen appears to enclose the most area?

3. Based on your sketches, how could you find the area of the pens without calculating them? Explain your reasoning.

4. Based on your answer to question 3, find the areas of the rectangular pens in the sketch.

5. Use your sketch of the rectangular pens to complete the data table below. Be sure to list pens that have the same dimensions but different corresponding lengths and widths. For example, include a 1 yd by 11 yd pen as well as an 11 yd by 1 yd pen.

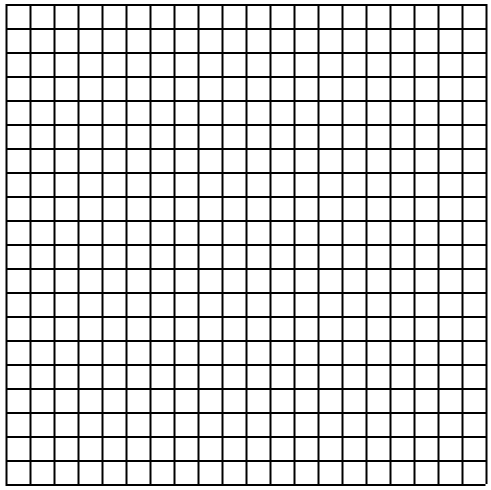
Length (yd)	**Width (yd)**	**Process for Perimeter**	**Perimeter (yd)**	**Process for Area**	**Area (yd^2)**
			24		
			24		
			24		
			24		
			24		
			24		
			24		
			24		
			24		
			24		
			24		

   a. According to the data in your table, describe how the width of the rectangle changes as the length increases.

   b. What does your answer to 5.a lead you to believe about the relationship between the width and length of a rectangle of perimeter 24?

Nelly’s on the Run! **(pp. 3 of 4)**

c. Using your graphing calculator and the data in your table, create a scatterplot of width vs. length for the rectangle of perimeter 24 yards. Sketch the scatterplot below.



d. Based on the data in your table and the scatterplot, write an equation for the width of the rectangle in terms of its length.

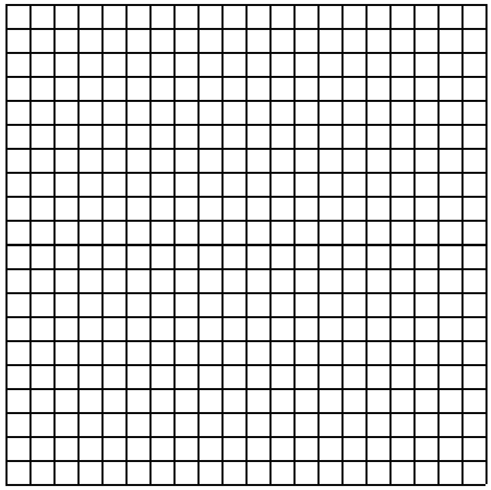
e. Enter the equation from 5.d as a function in the graphing calculator. Graph the function along with the scatterplot. Sketch the graph along with your scatterplot in 5.c.

f. Given the context of the problem, what is an appropriate domain and range for the function that relates width to length? Explain your reasoning in terms of the problem situation.

g. Suppose you wanted to use your function to construct rectangles of perimeter 24 yards with non-whole number side lengths. Find the width given a length of 7.25 yards. Find the length given a width of 6.6 yards. Write your answers as ordered pairs that satisfy your equation from 5.d.

Nelly's on the Run! (pp. 4 of 4)

6. Using your graphing calculator and the data in the table, create a scatterplot of area vs. length for the rectangle of perimeter 24 yards. Sketch the scatterplot below.



   a. Based on the data in your table, write an equation for the area in terms of the length of the rectangle of perimeter 24.

   b. Enter the equation from 6.a as a function in the graphing calculator. Graph the function along with the scatterplot. Sketch the graph along with your scatterplot in question 6.

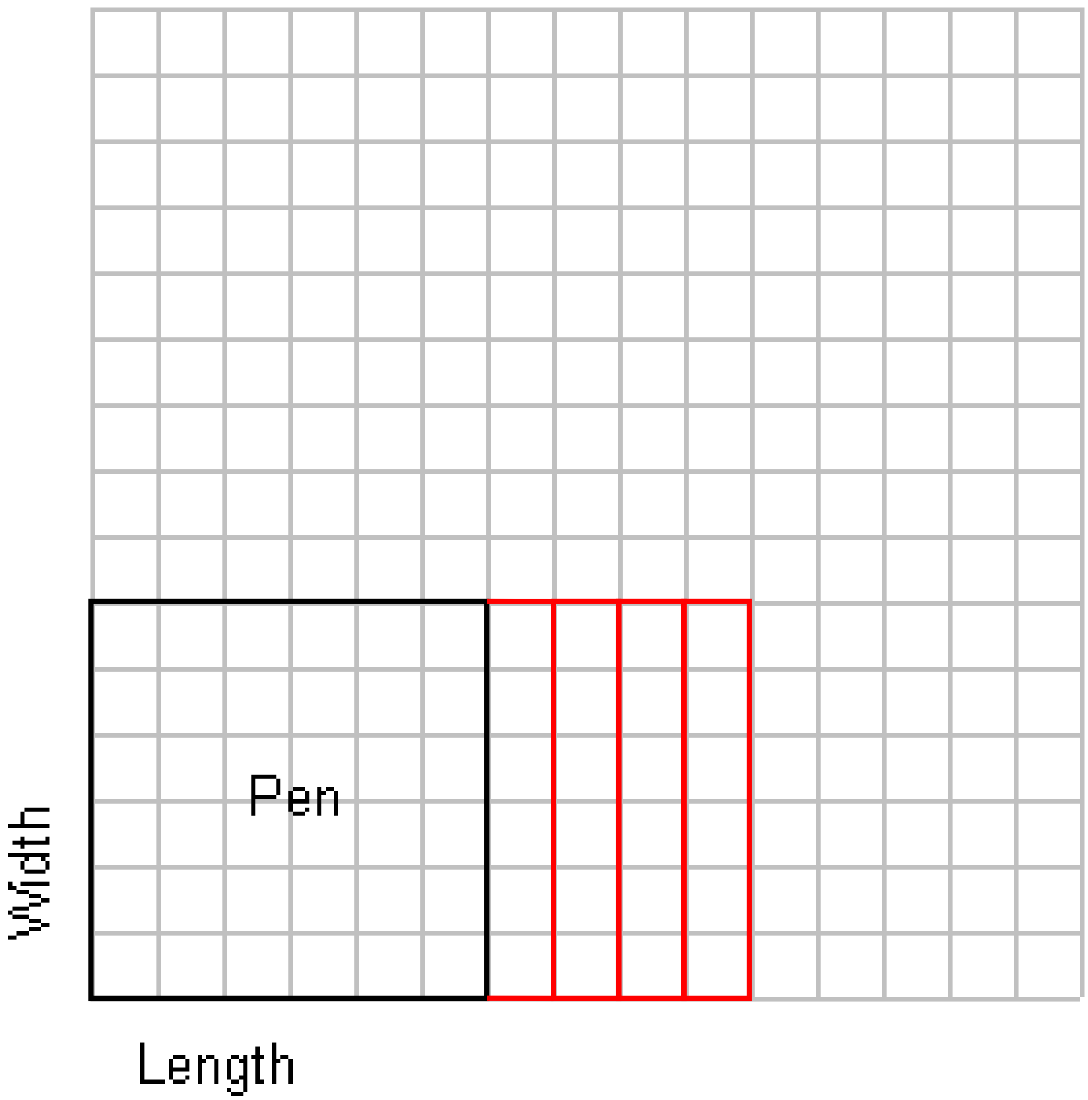
   c. Given the context of the problem, what is an appropriate domain and range for the function that relates area to length? Explain your reasoning in terms of the problem situation.

   d. Based on the area function, which rectangular pen will result in the most area? Is this the same or different from your answer in question 2? Explain.

Nelly's Got Company! (pp. 1 of 5) KEY

You constructed Nelly's pen to be 6 yards by 6 yards square since that would allow the maximum area given 24 yards of fencing. Now you are considering getting another goat, Buster, to give Nelly a little company and are considering ways to enlarge the pen.

1. Use the grid below to answer the following questions. Let each grid block represent 1 yd^2. Shown on the grid is the 6 yard by 6 yard square pen.



   a. Suppose you increase the length by 1 yard at a time. Use a colored pencil to sketch each additional area enclosed by increasing the length 1 yard, 2 yards, 3 yards, and 4 yards on the grid.

Nelly's Got Company! (pp. 2 of 5) KEY

b. Use your sketch from 1.a to complete the table below.

Increase in Length	Length (yd)	Width (yd)	Area (yd^2)	Perimeter (yd)
0	6	6	36	24
1	7	6	42	26
2	8	6	48	28
3	9	6	54	30
4	10	6	60	32

Examining Area:

c. According to the data in the table, by how much does the area of the rectangle change each time the length is increased by 1 yard? Explain.
Each time the length increased by 1 yard, the area increased by 6 square yards.

d. How is your answer to 1.c reflected in the sketches of the rectangular pens? Explain.
Increasing the length by 1 yard (the length of 1 grid block) resulted in a rectangular pen that enclosed 6 additional square yards (or 6 additional grid blocks).

e. Write an equation for the area of the pen if the length is increased by *x* yards.
$A = (6 + x)6$ or $A = 36 + 6x$

f. Use your equation from 1.e to find the area of the pen if the length is increased by 2.5 yards.
51 yd^2

g. Use your equation from 1.e to find the amount of increase in length needed to result in a pen whose area is 25% larger than the original.
1.5 yd

Examining Perimeter:

h. According to the data in the table, how much does the perimeter of the rectangle change each time the length is increased by 1 yard? Explain.
Each time the length increased by 1 yard, the perimeter increased by 2 yards.

i. How is your answer to 1.h reflected in the sketches of the rectangular pens? Explain.
Increasing the length by 1 yard (the length of 1 grid block) resulted in a rectangular pen whose perimeter was increased by 2 yards (the length of 2 grid blocks).

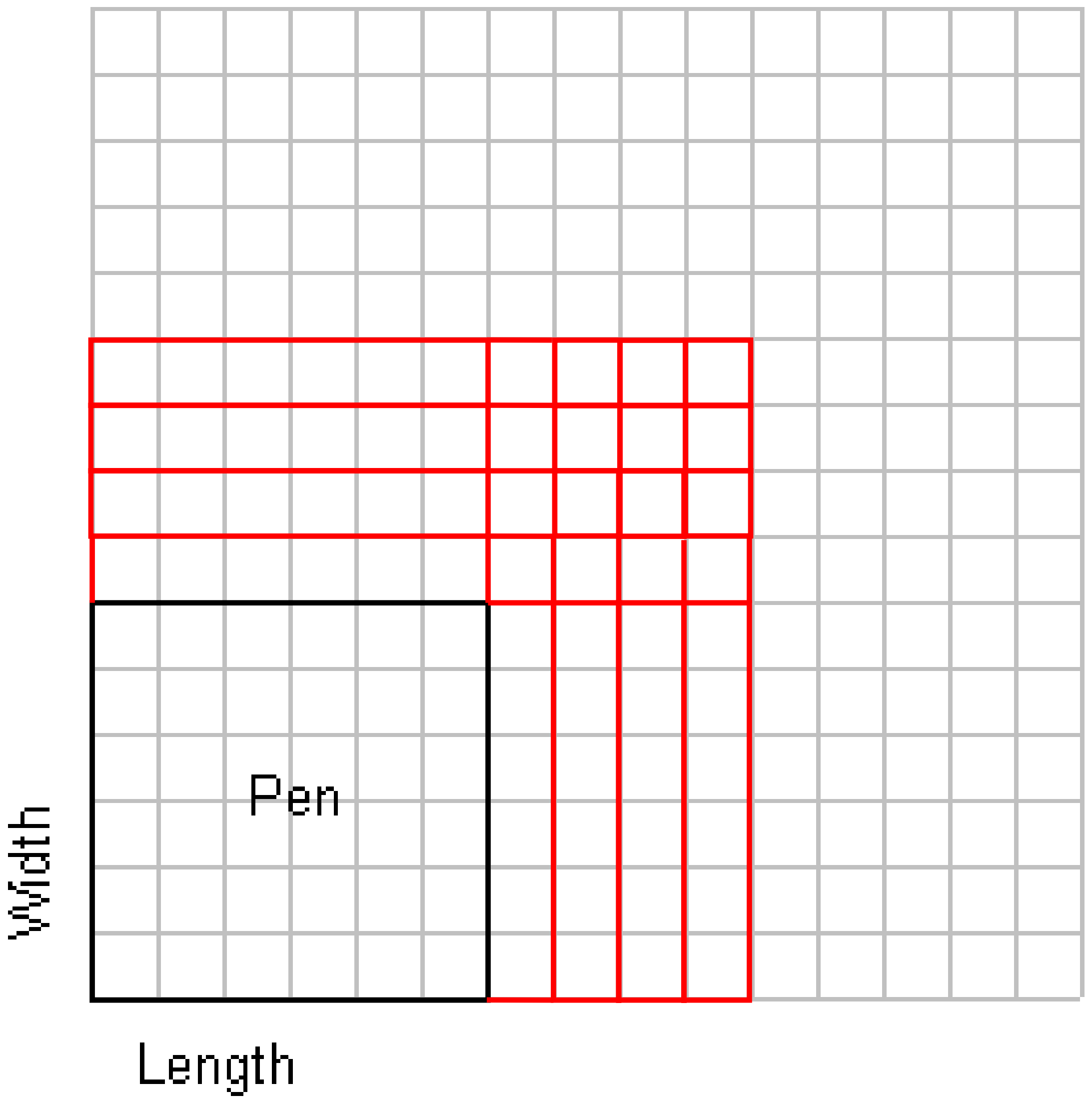
j. Write an equation for the perimeter of the pen if the length is increased by x yards.
$P = 2(6 + x + 6)$ or $P = 24 + 2x$

k. Use your equation from 1.j to find the perimeter of the pen if the length is increased by 2.5 yards.
29 yd

l. Use your equation from 1.j to find the amount of increase in length needed to result in a pen whose perimeter is 75% larger than the original.
Original + 0.75(original) = 24 + 0.75(24) = 42
$24 + 2x = 42$
9 yd

Nelly's Got Company! (pp. 3 of 5) KEY

2. Use the grid below to answer the following questions. Let each grid block represent 1 yd^2. Shown on the grid is the 6 yard by 6 yard square pen.



a. Suppose you increase the length *and* width by 1 yard at a time. Use a colored pencil to sketch each additional area enclosed by increasing the length *and* width 1 yard, 2 yards, 3 yards, and 4 yards on the grid.

Nelly's Got Company! (pp. 4 of 5) KEY

b. Use your sketch from 2.a to complete the table below.

Increase in Length and Width	Length (yd)	Width (yd)	Area (yd^2)	Perimeter (yd)
0	6	6	36	24
1	7	7	49	28
2	8	8	64	32
3	9	9	81	36
4	10	10	100	40

Examining Area:

c. According to the data in the table, how does the area of the rectangle change each time the length and width are increased by 1 yard? What does this lead you to believe about the relationship between the increase in length and width, and the area of the resulting figure? Explain.
The area does not change consistently each time the dimensions are increased by 1; therefore, the relationship between the increase in length and width and the resulting area is not linear.

d. How is your answer to 2.c reflected in the sketches of the rectangular pens? Explain.
Increasing the length and width by 1 yard (the length of 1 grid block) resulted in a rectangular pen that enclosed 6 additional square yards from increasing the length, 6 additional square yards from increasing the width, and 1 additional square yard from increasing both dimensions for a total increase of 13 square yards. Subsequent increases in both dimensions resulted in a rectangular pens that enclosed 15, 17, and 19 additional square yards respectively.

e. Write an equation for the area of the pen if the length and width are both increased by x yards.
$A = (6 + x)(6 + x)$ or $A = 36 + 12x + x^2$

f. Use your equation from 2.e to find the area of the pen if both the length and width are increased by 6 yards.
144 yd^2

g. Use your equation from 2.e to find the amount of increase in both length and width needed to result in a pen whose area is twice as large as the original.
$-6 + \sqrt{72} \approx 2.5 yd$

Examining Perimeter:

h. According to the data in the table, by how much does the perimeter of the rectangle change each time the length and width are increased by 1 yard? Explain.
Each time the length and width increase by 1 yard, the perimeter increases by 4 yards.

Nelly's Got Company! (pp. 5 of 5) KEY

i. How is your answer to 2.h reflected in the sketches of the rectangular pens? Explain.
Increasing the length and width by 1 yard (the length of 1 grid block) results in a rectangular pen whose perimeter increases by 4 yards (the length of 4 grid blocks).

j. Write an equation for the perimeter of the pen if the length is increased by x yards.
$P = 2(6 + x + 6 + x)$ or $P = 24 + 4x$

k. Use your equation from 2.j to find the perimeter of the pen if the length and width are increased by 3.5 yards.
38 yd

l. Use your equation from 2.j. to find the amount of increase in length and width needed to result in a pen whose perimeter is 125% larger than the original.
Original + 1.25(original) = 24 + 1.25(24) = 54 yd
$24 + 4x = 54$
7.5 yd

3. Based on your findings, summarize how changing one or both dimensions of a rectangular figure affect perimeter and area measures for the figure.

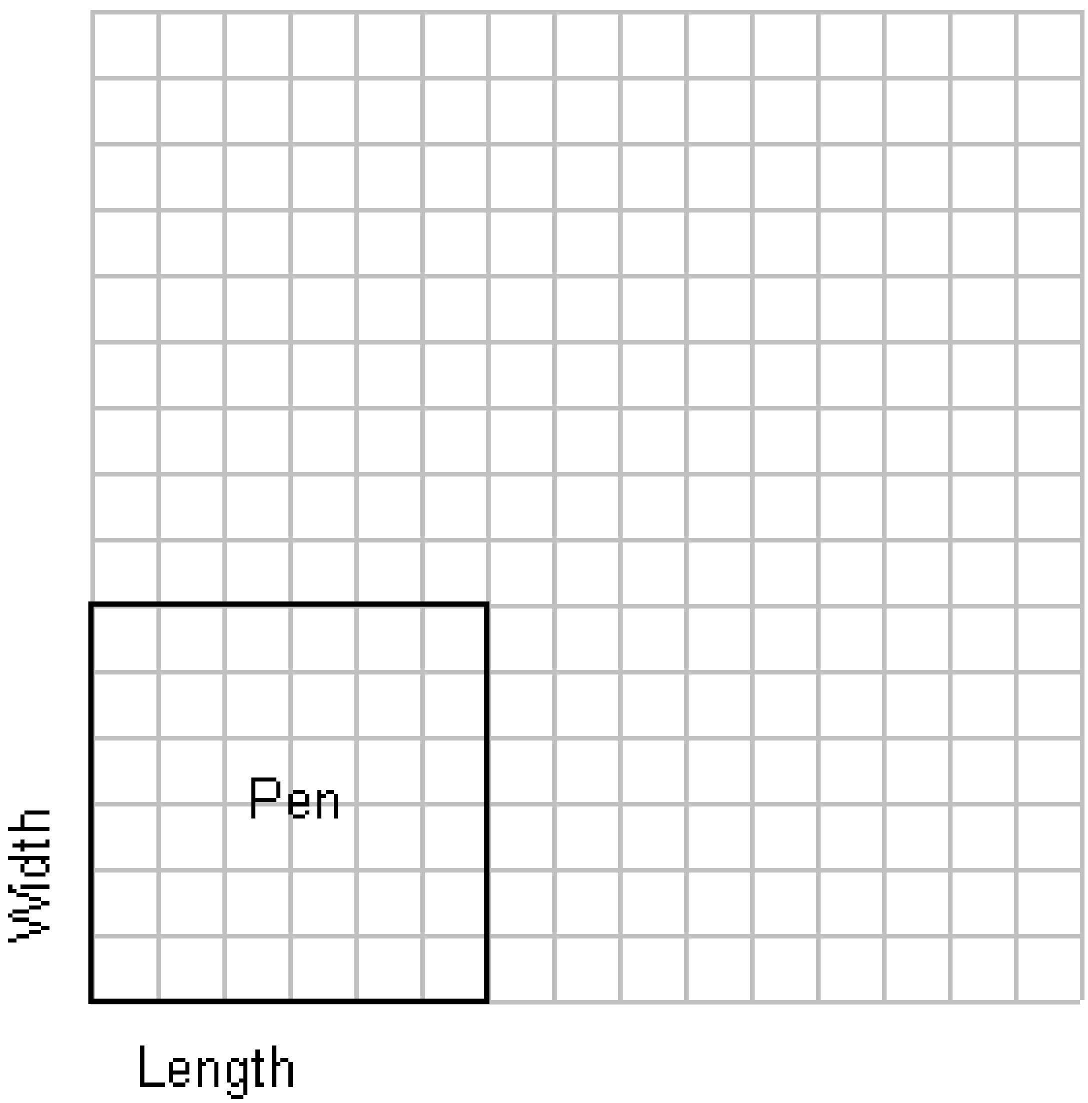
Changing one dimension of the figure causes both the perimeter and area to change linearly.

Changing both dimensions of the figure causes the perimeter to change linearly, while the area changes in a non-linear fashion.

Nelly's Got Company! (pp. 1 of 5)

You constructed Nelly's pen to be 6 yards by 6 yards square since that would allow the maximum area given 24 yards of fencing. Now you are considering getting another goat, Buster, to give Nelly a little company and are considering ways to enlarge the pen.

1. Use the grid below to answer the following questions. Let each grid block represent 1 yd^2. Shown on the grid is the 6 yard by 6 yard square pen.



   a. Suppose you increase the length by 1 yard at a time. Use a colored pencil to sketch each additional area enclosed by increasing the length 1 yard, 2 yards, 3 yards, and 4 yards on the grid.

Nelly's Got Company! (pp. 2 of 5)

b. Use your sketch from 1.a to complete the table below.

Increase in Length	Length (yd)	Width (yd)	Area (yd^2)	Perimeter (yd)
0	6	6	36	24
1		6		
2		6		
3		6		
4		6		

Examining Area:

c. According to the data in the table, by how much does the area of the rectangle change each time the length is increased by 1 yard? Explain.

d. How is your answer to 1.c reflected in the sketches of the rectangular pens? Explain.

e. Write an equation for the area of the pen if the length is increased by x yards.

f. Use your equation from 1.e to find the area of the pen if the length is increased by 2.5 yards.

g. Use your equation from 1.e to find the amount of increase in length needed to result in a pen whose area is 25% larger than the original.

Examining Perimeter:

h. According to the data in the table, how much does the perimeter of the rectangle change each time the length is increased by 1 yard? Explain.

i. How is your answer to part h reflected in the sketches of the rectangular pens? Explain.

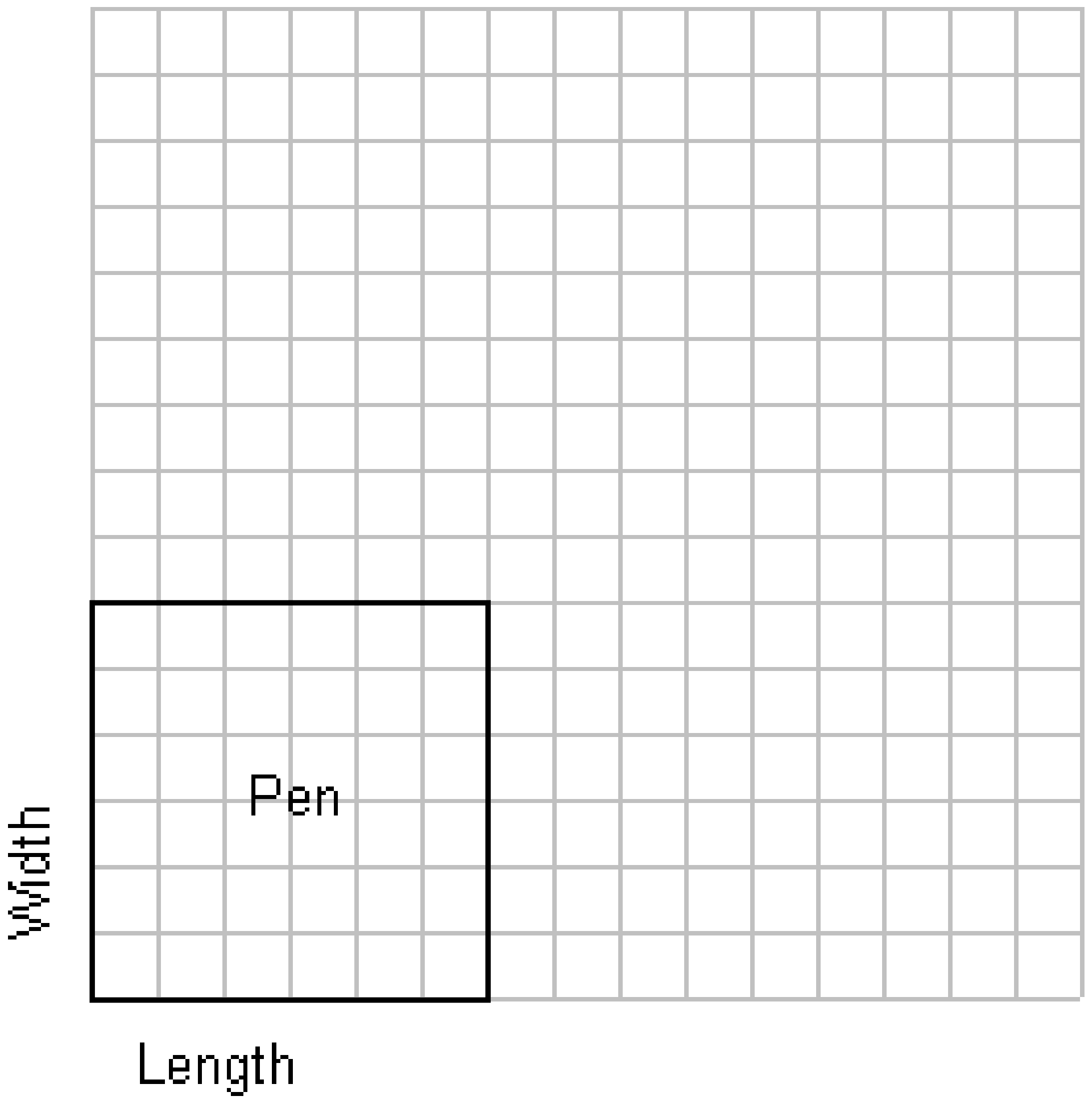
j. Write an equation for the perimeter of the pen if the length is increased by *x* yards.

k. Use your equation from 1.j to find the perimeter of the pen if the length is increased by 2.5 yards.

l. Use your equation from 1.j to find the amount of increase in length needed to result in a pen whose perimeter is 75% larger than the original.

Nelly's Got Company! (pp. 3 of 5)

2. Use the grid below to answer the following questions. Let each grid block represent 1 yd^2. Shown on the grid is the 6 yard by 6 yard square pen.



   a. Suppose you increase the length *and* width by 1 yard at a time. Use a colored pencil to sketch each additional area enclosed by increasing the length *and* width 1 yard, 2 yards, 3 yards, and 4 yards on the grid.

Nelly's Got Company! (pp. 4 of 5)

b. Use your sketch from 2.a to complete the table below.

Increase in Length and Width	**Length (yd)**	**Width (yd)**	**Area (yd^2)**	**Perimeter (yd)**
0	6	6	36	24
1				
2				
3				
4				

Examining Area:

c. According to the data in the table, how does the area of the rectangle change each time the length and width are increased by 1 yard? What does this lead you to believe about the relationship between the increase in length and width, and the area of the resulting figure? Explain.

d. How is your answer to 2.c reflected in the sketches of the rectangular pens? Explain.

e. Write an equation for the area of the pen if the length and width are both increased by x yards.

f. Use your equation from 2.e to find the area of the pen if both the length and width are increased by 6 yards.

g. Use your equation from 2.e to find the amount of increase in both length and width needed to result in a pen whose area is twice as large as the original.

Examining Perimeter:

h. According to the data in the table, by how much does the perimeter of the rectangle change each time the length and width are increased by 1 yard? Explain.

Nelly's Got Company! (pp. 5 of 5)

i. How is your answer to 2.h reflected in the sketches of the rectangular pens? Explain.

j. Write an equation for the perimeter of the pen if the length is increased by *x* yards.

k. Use your equation from 2.j to find the perimeter of the pen if the length and width are increased by 3.5 yards.

l. Use your equation from 2.j. to find the amount of increase in length and width needed to result in a pen whose perimeter is 125% larger than the original.

3. Based on your findings, summarize how changing one or both dimensions of a rectangular figure affect perimeter and area measures for the figure.

Ring Around the Roses (pp. 1 of 3)

Background:
The circumference of a circle is a function of its diameter. The area of a circle is a function of a power of its radius. In this experiment you will collect data to develop these two relationships.

Objectives:
Use concrete models to formulate and test conjectures about the properties and attributes of circles.

Materials:
Card stock, safety compass, ruler, beans, cereal, or candy

Procedure: (3-4 group)

Part A: Circumference

1. On one side of the card stock, draw circles with radii of 1, 2, and 3 inches. On the other side draw circles with radii of 4 inches and ½ inch.
2. Draw a line for the diameter of each circle.
3. Begin with the ½ inch circle. Lay a row of beans, cereal, or candy along the diameter. Record the number of beans, cereal, or candy required for the diameter in the table.
4. Go around the circle using beans, cereal, or candy. Record the number of beans, cereal, or candy required to go around the circle in the table.
5. Continue this process for the remaining circles.

Circle	Beans across diameter (d)	Beans around circle, circumference (C)	Point (d, C)
½ inch			
1 inch			
2 inch			
3 inch			
4 inch			

Ring Around the Roses (pp. 2 of 3)

Part B: Area

1. Begin with the ½ inch circle. Lay a row of beans, cereal, or candy along the radius (halfway across). Record the number of beans, cereal, or candy required for the radius in the table.
2. Cover the inside of the circle with beans, cereal, or candy. Record the number of beans, cereal, or candy required to cover the inside of the circle in the table.
3. Continue this process for the remaining circles.

Circle	Beans across radius (r)	Beans inside Circle, area (A)	Point (r, A)
½ inch			
1 inch			
2 inch			
3 inch			
4 inch			

<u>Results</u>:

Part A: Circumference

1. Which is the independent and dependent variable?

2. Graph a scatter plot of circumference versus diameter. Scale and label axes.

3. Adjust the linear parent function to determine a trend line for the data.

4. Graph the line on the scatter plot and label.

Part B: Area

1. Which is the independent and dependent variable?

2. Graph a scatter plot of radius versus area. Scale and label axes.

3. Adjust the quadratic parent function to determine a trend quadratic function for the data.

4. Graph the parabola on the scatter plot and label.

Ring Around the Roses (pp. 3 of 3)

Conclusions:

1. For the scatter plot of the circumference, what was the shape of the scatter plot?

2. What was the trend line for the data on circumference? How does this compare to the actual function $C = \pi d$, where $\pi = 3.14$?

3. Using your trend line as the observed value and 3.14 as the actual value, find your percent error.

$$\text{Percent error} = \frac{|\text{actual - observed}|}{\text{actual}} \times 100$$

4. The area is a direct variation of a power of the radius. What was the shape of the scatter plot for area versus radius?

5. The actual function is $A = \pi r^2$. Calculate the percent error on the “a” value you observed when you found the trend quadratic function for the data.

6. Was your percent error greater for the circumference or area formula? Explain reasons for the error.

Going Around in Circles Teacher Notes (pp. 1 of 3)

Objective:

Explore the relationship between diameter, radius, and circumference in order to discover the value and meaning of pi, π.

Materials:

Circular objects, ribbon, scissors, meter stick, chart paper, markers, tape or glue stick, sticky dots, graphing calculator

Procedure:

1. Put students in pairs. Give each pair two circular objects and two strips of ribbon or string (enough to go around the circular objects).

2. Demonstrate how to measure the circumference with the ribbon and cut off a strip of ribbon representing the circumference. Discuss with students how each strip of ribbon is associated with one circular object.

3. Have students use a meter stick to measure the radius, diameter, and circumference of the two circular objects. Ask students how they can use the meter stick to measure the curved circumference, i.e., use the piece of ribbon they cut. Use units as applicable by content area and grade level.

Group Circle Data

Radius	Diameter	Circumference

4. Have all students put their results on a posted class data table.

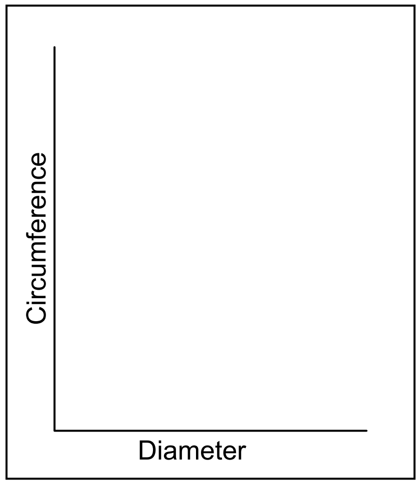
Class Circle Data

Radius	Diameter	Circumference
0	0	0

Going Around in Circles Teacher Notes (pp. 2 of 3)

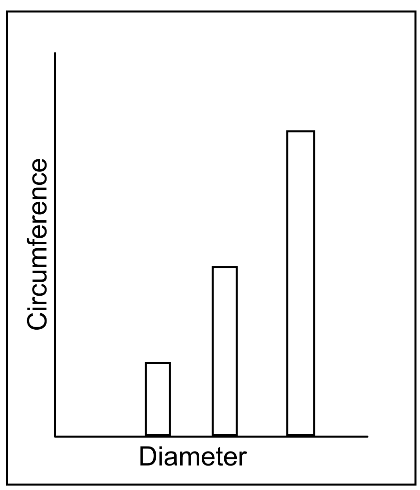
5. Post a sheet of chart paper on the board or wall of the classroom. Place an "L" shaped axis on the grid. Label the horizontal axis as diameter. Label the vertical axis as circumference.

Graph of Circle Relationships



6. Have each group put their strips of ribbon on the chart paper with tape or glue stick. Using the circular object, students should measure the diameter on the horizontal axis, and attach the ribbon vertically at that point on the horizontal axis.

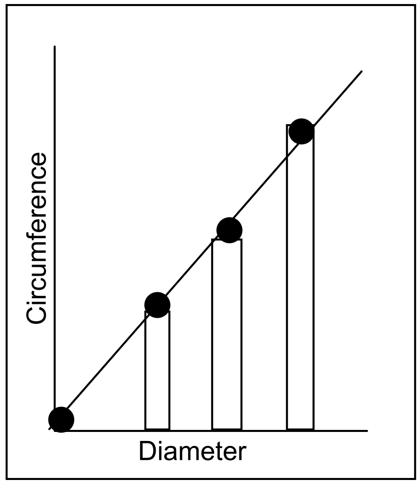
Graph of Circle Relationships



Going Around in Circles Teacher Notes (pp. 3 of 3)

7. Place sticky dots at the top of each strip of ribbon. Also place a sticky dot at the origin, (0, 0). Use a strip of black ribbon to illustrate a line of best fit through the points.

Graph of Circle Relationships

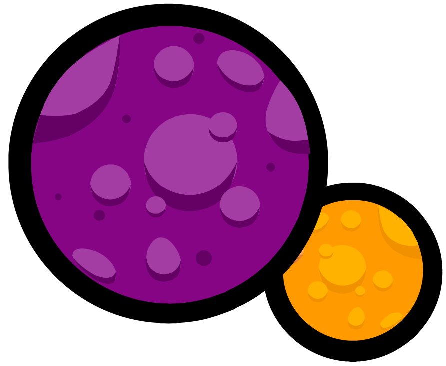


8. Have students draw a sketch of the Graph of Circle Relationships on their paper, and write a paragraph describing what they notice about the graph.

9. Have students complete the “Results and Conclusions” section of the student sheet individually.

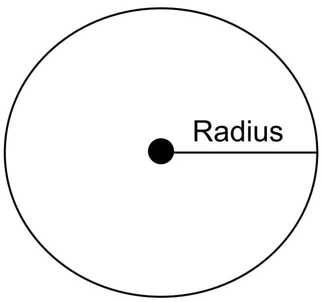
10. When completed, discuss “Results and Conclusions” as a class, encouraging students share what they have learned.

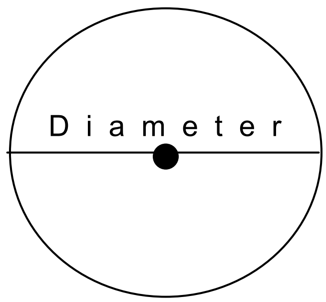
Going Around in Circles (pp. 1 of 5)

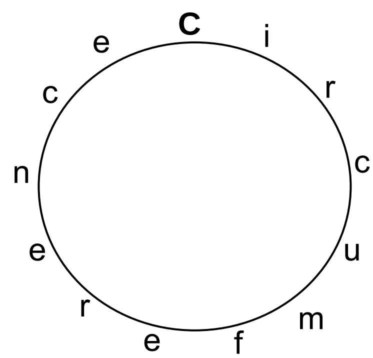


Background:

- Radius – distance from the circle to the center of the circle, equal to ½(diameter)
- Diameter – distance across a circle passing through the center of the circle, equal to 2(radius)
- Circumference – distance around the circle







Objective:

Explore the relationship between diameter, radius, and circumference in order to discover the value and meaning of pi, π .

Procedure:

1. Working in pairs, represent the circumference of two circular objects by cutting off strips of ribbon that are the lengths of the circumferences.
2. Estimate and mark a point on each circular object to represent the center of the circle. Use a meter stick to measure the radius, diameter, and circumference of the circular objects. Record the results in the table below.

Group Data Chart

Radius	Diameter	Circumference

3. Record your group data on the Class Data Chart. Why is the first row in the chart all zeros?

Going Around in Circles (pp. 2 of 5)

4. Place your strips of ribbon representing the length of the circumference on the chart "Graph of Circle Relationships" with tape or glue stick. Use the circular object to measure the diameter on the horizontal axis and at that point attach the ribbon vertically on the horizontal. Place a sticky dot at the top of the ribbon.

5. When all ribbons have been attached, draw a sketch below. Title the graph and label the axes. Describe what you notice in the graph.

Describe:

Going Around in Circles (pp. 3 of 5)

Results and Conclusions:

1. Make a data table comparing diameter and circumference from the Class Data Chart.

Diameter (d)	Circumference (C)
0	0

2. Use the patterns to develop a rule (formula) for circumference in terms of diameter.

Remember:

$$\mathbf{y} = \frac{\text{change in output}}{\text{change in input}}\ \mathbf{x}\ \ +\ \ \text{starting point at zero}$$

3. How would the rule (formula) for circumference in terms of radius be written? (Hint: Use the fact that $d = 2r$.)

4. What does the graph of the relationship between the circumference and diameter look like?

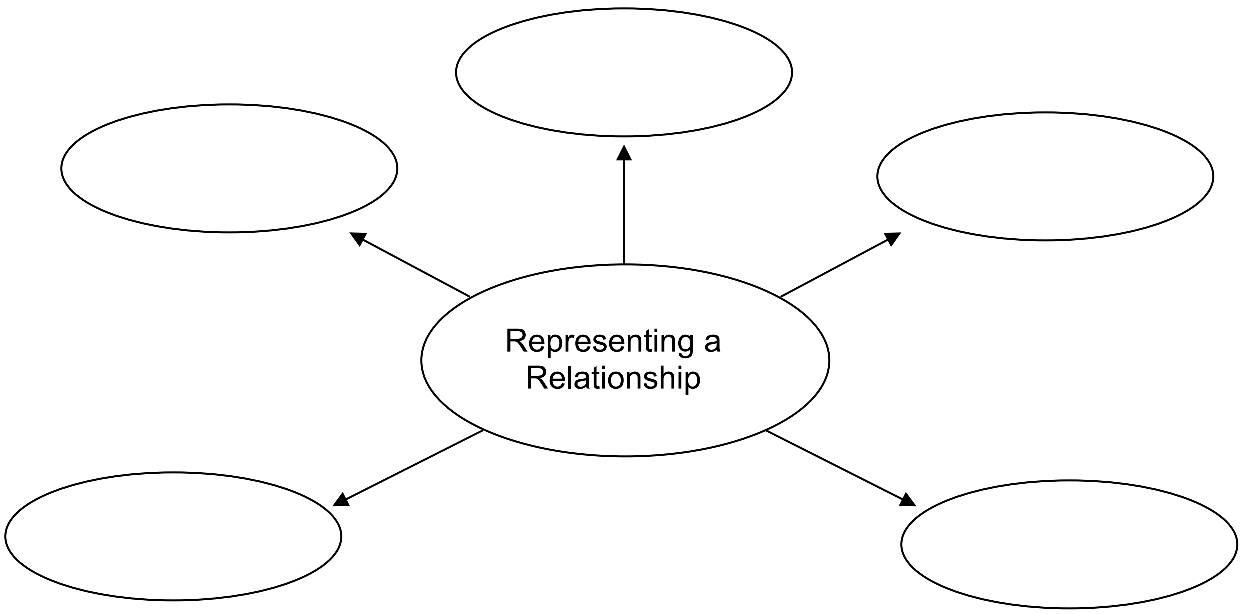
5. What is the rate of change in the graph? Where is this found in the formula?

Going Around in Circles (pp. 4 of 5)

6. Write a verbal description of the relationship between circumference and diameter.

7. Is the relationship proportional? Explain.

8. Complete a concept map to illustrate at least five different ways the relationship between two variables can be represented.



9. Enter the data for diameter and circumference into the graphing calculator in L_1 and L_2. Make a scatterplot of the data. Enter your rule (formula) into y^1. How closely does your rule fit the data points?

Going Around in Circles (pp. 5 of 5)

10. Use the developed rule and the graphing calculator to predict the circumference of circles with a diameter of
    a. 25 units

    b. 130 units

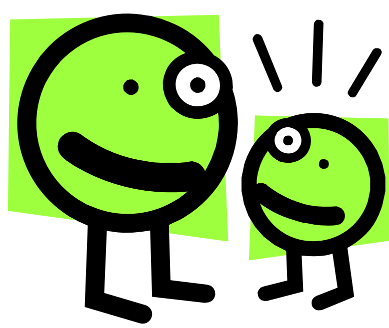
    c. 475 units

11. Use the developed rule and graphing calculator to predict the diameter of circles with a circumference of
    a. 85 units

    b. 258 units

    c. 844 units

12. Explain how data collection can be used to make predictions.



Perimeter, Circumference, and Area (pp. 1 of 3) KEY

Notes

The perimeter of a polygon is the total distance around a polygon. Although there are formulas for finding the perimeter of a rectangle (2*l* + 2*w*) and for the square (4*s*), perimeter for any polygon can be found by adding together all the sides of the figure.

The circumference of a circle is the total distance around a circle. There are two formulas for finding the circumference of a circle (πd and 2πr).

The area of polygons and circles is the surface covered within the figure. Formulas for the area of polygons depend on the type of polygon. The formula for the area of a circle is always πr^2.

Below are the formulas for perimeter, circumference, and area as they appear on the TAKS Mathematics Chart.

Grades 9, 10, and Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	regular polygon	$A = \frac{1}{2}aP$
	circle	$A = \pi r^2$

P represents the Perimeter of the Base of a three-dimensional figure.

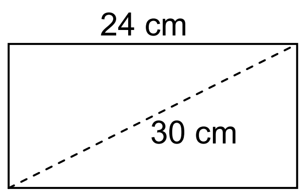
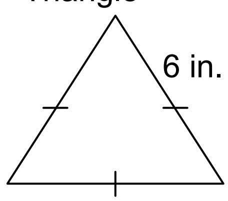
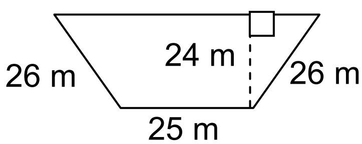
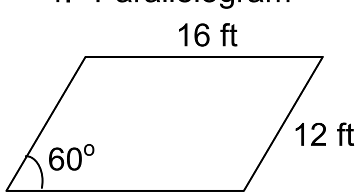
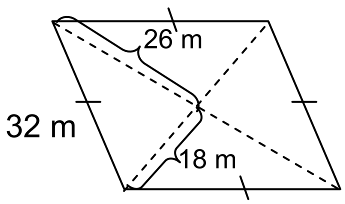
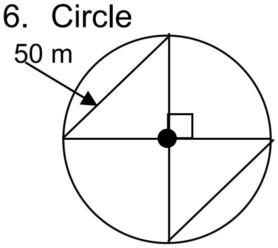
B represents the Area of the Base of a three-dimensional figure.

When one dimension is changed:
- Perimeter is multiplied by the scale factor of the dimension.
- Area is multiplied by the scale factor of the dimension.

When two dimensions are changed:
- Perimeter is multiplied by the scale factor of the dimensions.
- Area is multiplied by the square of the scale factor of the dimensions.

Perimeter, Circumference, and Area (pp. 2 of 3) KEY

Practice Problems (Round answers to the nearest hundredth, when necessary.)

Figure	Perimeter/Circumference	Area
1. Rectangle 	84 cm	432 cm^2
2. Triangle 	18 in	15.59 in^2
3. Trapezoid 	122 m	840 m^2
4. Parallelogram 	56 ft	166.28 ft^2
5. Rhombus 	128 m	936 m^2
6. Circle 	222.14 m	3,926.99 m^2

Perimeter, Circumference, and Area (pp. 3 of 3) KEY

Draw a diagram and find the solution to the questions. Round answers to the nearest hundredth.

7. A garden is in the shape of an isosceles trapezoid with legs of 15 feet and bases of 10 feet and 28 feet. How many feet of fencing will be needed to enclose the garden? If all the area can be utilized, how many square feet of garden will be available for planting?
   Fencing: 68 ft
   Planting Area: 228 ft^2

8. After a release at one of the chemical plants, warnings were sent out to all areas within a 3.2 mile radius to “Shelter in Place”. How many square miles had to “Shelter in Place”? How many square kilometers had to “Shelter in Place”? (1 mile = 1.61 kilometers)
   32.17 square miles had to shelter in place.
   83.39 square kilometers had to shelter in place.

9. Sue Ann is cutting out felt pieces in the shape of parallelograms with consecutive sides of 12 cm and 18 cm and with base angles of 30^o and 150^o. Each will be trimmed with a strip of rickrack. How long a strip of rickrack in centimeters will be needed to trim each felt piece? How many square centimeters of felt will be need for each piece? How many inches of rickrack and square inches of felt will be needed? (1 inch = 2.54 centimeters)
   Rickrack: 60 cm; Area of Felt: 108 cm^2
   Rick rack: 23.6 inches; Area of Felt: 16.74 in.^{2}

10. The lid for a lead drum to hold nuclear waste has a diameter of 3.2 feet. It weighs 6.5 pounds per square foot. How much does the lid to the drum weigh?
    The lid would be 8.04 square feet with a weight of 52.28 pounds.

11. A square bathroom window has diagonals of 42 inches. How many feet of trim will be needed for the frame? How many square feet of glass will be needed for the window pane?
    Trim for the frame: 118.8 in
    Area of glass: 882 in^2

12. Papa’s Pizza charges $0.05 per square inch of pizza. Pizza is measured by the diameter. How much will Papa’s Pizza charge for a 7-inch, 10-inch, and 14-inch pizza? From your results, do you think this is how the price of pizza is determined? Explain.
    7-inch pizza, $A = \pi(7)^2$ = 153.94 square inches, which would cost $7.70
    10-inch pizza, $A = \pi(10)^2$ = 314.16 square inches, which would cost $15.71
    14-inch pizza, $A = \pi(14)^2$ = 615.75 square inches, which would cost $30.79
    This is not how it would be determined because large pizzas would be too high to purchase.

13. Mary Ann purchased a new circular table for her kitchen that is similar to the original circular table but only $\frac{4}{5}$ the original size. The circumference of the original table was 440 centimeters. The area the original table covered was 15,394 square centimeters. What would be the circumference and area covered by the new circular table?
    Circumference of the new circular table = 352 centimeters
    Area of the new circular table = 9,852.16 square centimeters

Perimeter, Circumference, and Area (pp. 1 of 3)

Notes

The ____________________ of a polygon is the total distance around a polygon. Although there are formulas for finding the perimeter of a rectangle ($2l + 2w$) and for the square ($4s$), perimeter for any polygon can be found by adding together all the sides of the figure.

The ____________________ of a circle is the total distance around a circle. There are two formulas for finding the circumference of a circle (πd and $2\pi r$).

The ____________ of polygons and circles is the surface covered within the figure. Formulas for the area of polygons depend on the type of polygon. The formula for the area of a circle is always πr^2.

Below are the formulas for perimeter, circumference, and area as they appear on the TAKS Mathematics Chart.

Grades 9, 10, and Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	regular polygon	$A = \frac{1}{2}aP$
	circle	$A = \pi r^2$

P represents the Perimeter of the Base of a three-dimensional figure.

B represents the Area of the Base of a three-dimensional figure.

When one dimension is changed:

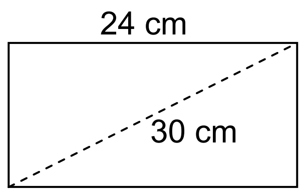
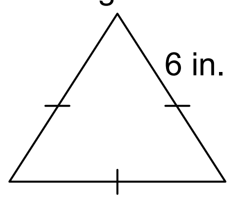
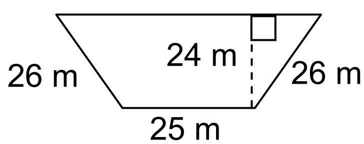
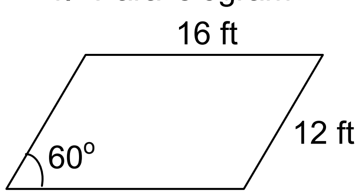
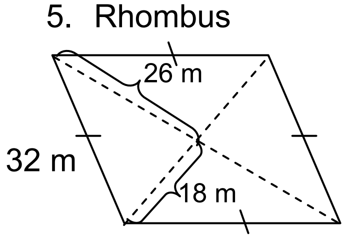
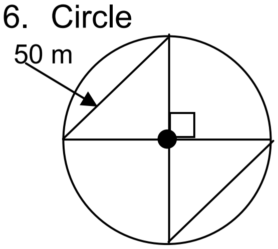
- Perimeter is multiplied by the ____________________ of the dimension.
- Area is multiplied by the ____________________ of the dimension.

When two dimensions are changed:

- Perimeter is multiplied by the ____________________ of the dimensions.
- Area is multiplied by the ______________________________ of the dimensions.

Perimeter, Circumference, and Area (pp. 2 of 3)

Practice Problems (Round answers to the nearest hundredth, when necessary.)

Figure	Perimeter/Circumference	Area
1. Rectangle 		
2. Triangle 		
3. Trapezoid 		
4. Parallelogram 		
5. Rhombus 		
6. Circle 		

Perimeter, Circumference, and Area (pp. 3 of 3)

Draw a diagram and find the solution to the questions. Round answers to the nearest hundredth.

7. A garden is in the shape of an isosceles trapezoid with legs of 15 feet and bases of 10 feet and 28 feet. How many feet of fencing will be needed to enclose the garden? If all the area can be utilized, how many square feet of garden will be available for planting?

8. After a release at one of the chemical plants, warnings were sent out to all areas within a 3.2 mile radius to “Shelter in Place”. How many square miles had to “Shelter in Place”? How many square kilometers had to “Shelter in Place”? (1 mile = 1.61 kilometers)

9. Sue Ann is cutting out felt pieces in the shape of parallelograms with consecutive sides of 12 cm and 18 cm and with base angles of 30^{o} and 150^{o}. Each will be trimmed with a strip of rickrack. How long a strip of rickrack in centimeters will be needed to trim each felt piece? How many square centimeters of felt will be need for each piece? How many inches of rickrack and square inches of felt will be needed? (1 inch = 2.54 centimeters)

10. The lid for a lead drum to hold nuclear waste has a diameter of 3.2 feet. It weighs 6.5 pounds per square foot. How much does the lid to the drum weigh?

11. A square bathroom window has diagonals of 42 inches. How many feet of trim will be needed for the frame? How many square feet of glass will be needed for the window pane?

12. Papa’s Pizza charges $0.05 per square inch of pizza. Pizza is measured by the diameter. How much will Papa’s Pizza charge for a 7-inch, 10-inch, and 14-inch pizza? From your results, do you think this is how the price of pizza is determined? Explain.

13. Mary Ann purchased a new circular table for her kitchen that is similar to the original circular table but only $\frac{4}{5}$ the original size. The circumference of the original table was 440 centimeters. The area the original table covered was 15,394 square centimeters. What would be the circumference and area covered by the new circular table?

Measuring Composite Figures (pp. 1 of 3) KEY

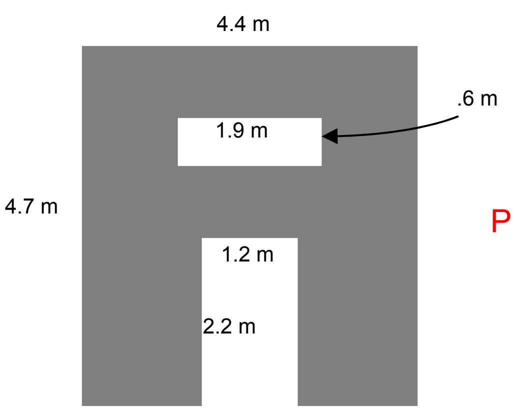
A composite figure is made up of multiple shapes. In order to find the perimeter add all the individual sides. In order to find the area, divide the composite figure into several smaller figures for which the areas can be found using formulas. Add all the individual areas together.

1. If necessary, measure each figure in centimeters. Find the perimeter and area of the shaded region of the composite figures.

a.

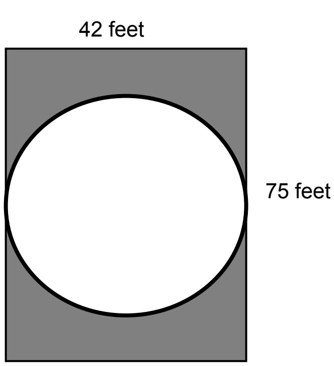
P = 20.1 cm, A = 18.41 cm^2

b.



P = 27.6 m, A = 16.9 m^2

c.

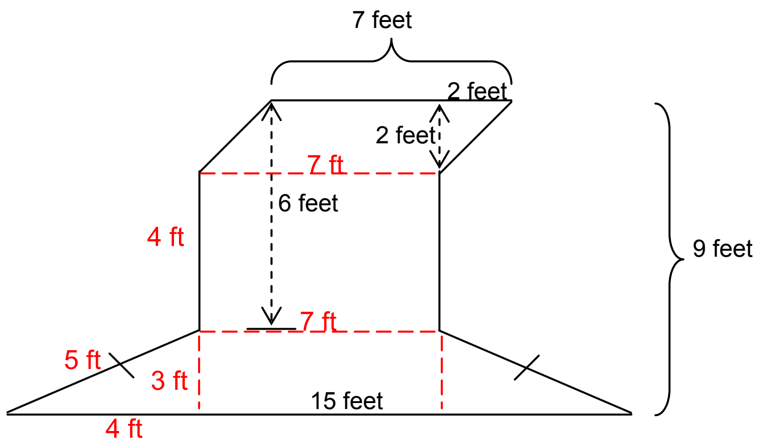


P = 2(42) + 2(75) + π(42)
P = 365.95 ft

(42)(75) - π(42/2)^2
A = 1,764.56 ft^2

Measuring Composite Figures (pp. 2 of 3) KEY

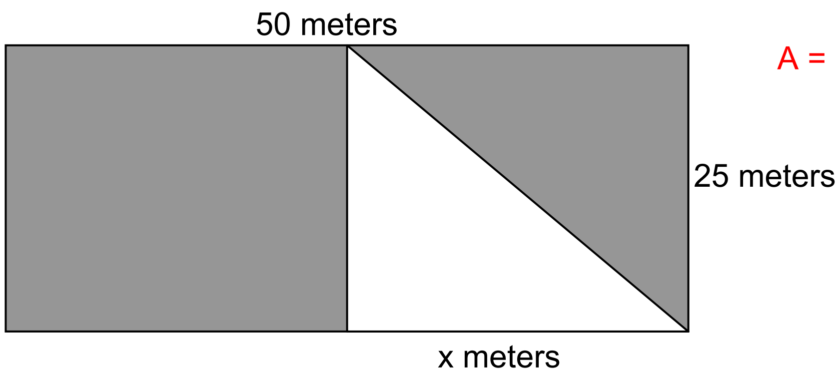
2. Tomas wants to put a Japanese garden in his backyard with the following dimensions. How many feet of wooden walkway will he have to buy to surround the garden? How many square feet will the garden alone take up?



P = 45.6 ft., A = 75 ft.^{2}

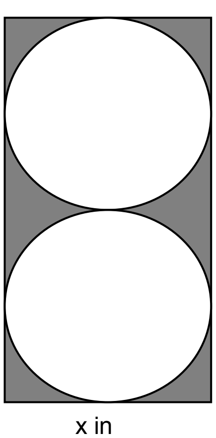
3. What expression could be used to represent the area of the shaded regions?

a.



A = 1250 – 12.5x

b.



$A = 2x^2 - \frac{1}{2}\pi x^2$

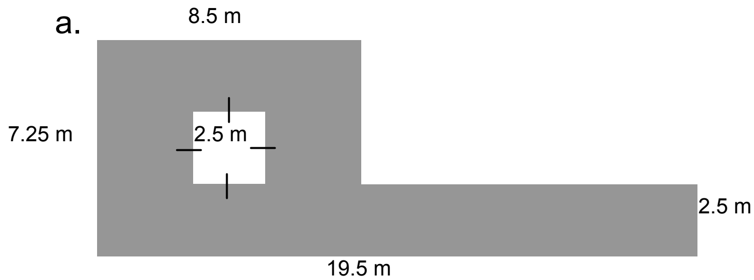
or

$A = \frac{4x^2 - \pi x^2}{2}$

Measuring Composite Figures (pp. 3 of 3) KEY

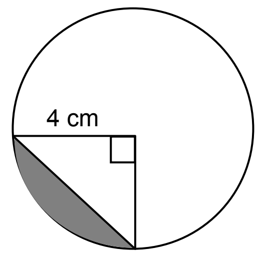
4. Find the perimeter and area of the shaded region of the composite figure. Round to the nearest hundredth when necessary.

a.



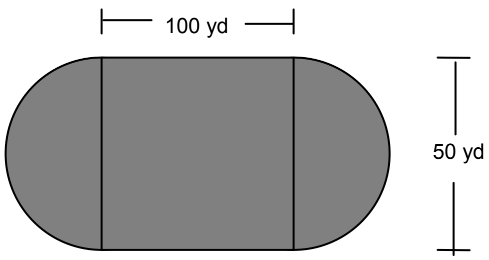
P = 63.5 m, A = 82.875 m^2

b.



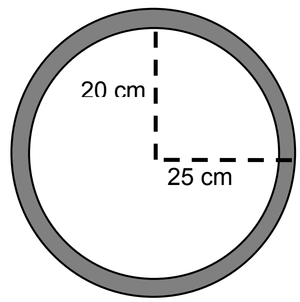
P = 11.94 cm, A = 4.57 cm^2

c.



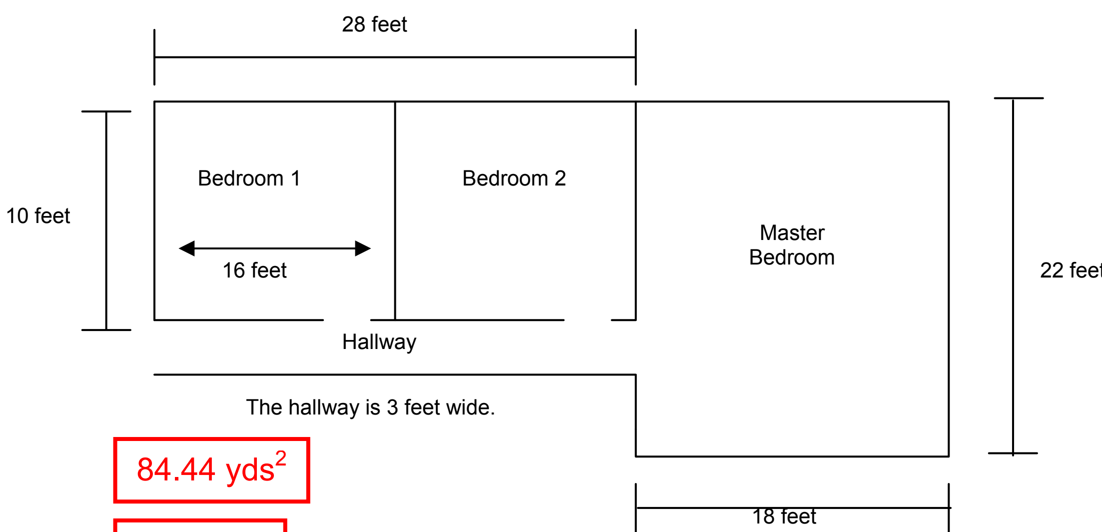
P = 357.08 yd, A = 6963.50 yd^2

d.



P = 282.74 cm, A = 706.86 cm^2

5. Jill is planning on carpeting the hall and bedrooms in her house. The floor plan is below. Find the amount of carpet she will need in square yards. Find the amount in square meters. (1.09 yards = 1 meter)



84.44 yds^2

71.07 m^2

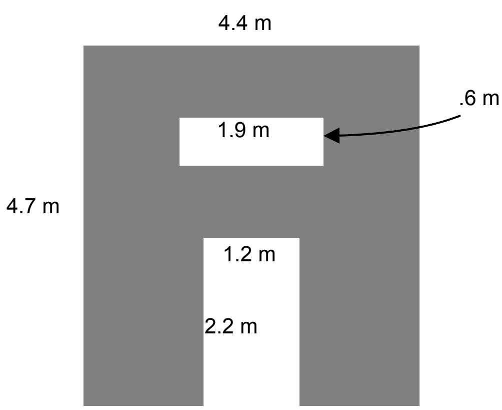
Measuring Composite Figures (pp. 1 of 3)

A composite figure is made up of multiple shapes. In order to find the perimeter add all the individual sides. In order to find the area, divide the composite figure into several smaller figures for which the areas can be found using formulas. Add all the individual areas together.

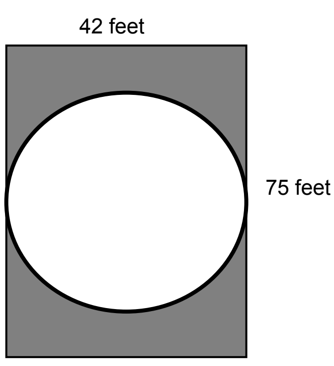
1. If necessary, measure each figure in centimeters. Find the perimeter and area of the shaded region of the composite figures.

a.

b.

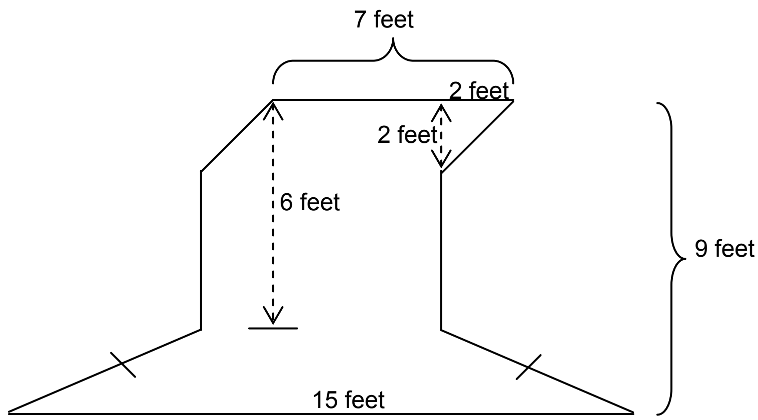


c.



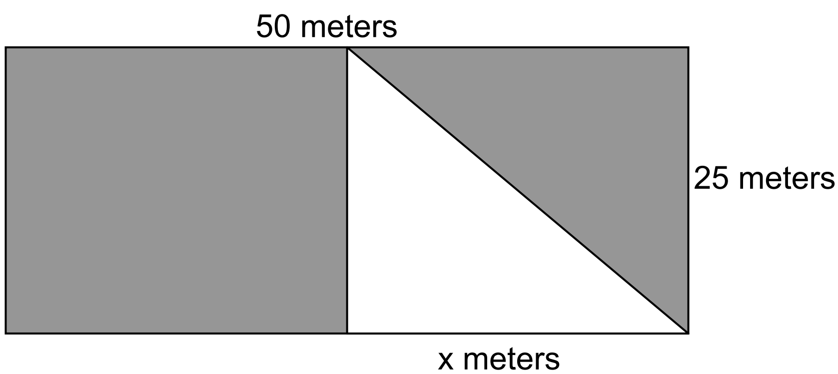
Measuring Composite Figures **(pp. 2 of 3)**

2. Tomas wants to put a Japanese garden in his backyard with the following dimensions. How many feet of wooden walkway will he have to buy to surround the garden? How many square feet will the garden alone take up?

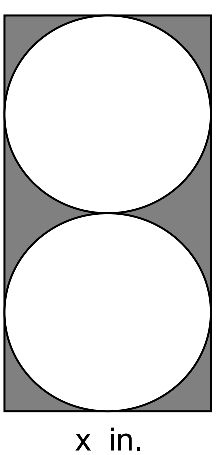


3. What expression could be used to represent the area of the shaded regions?

a.



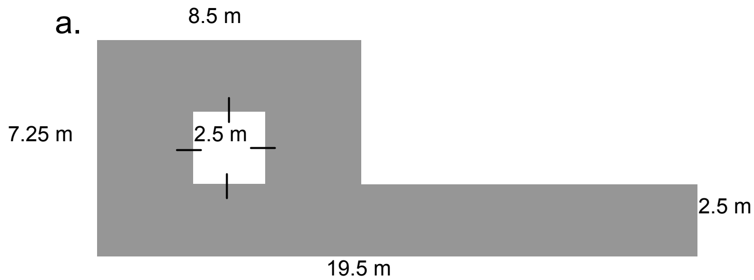
b.



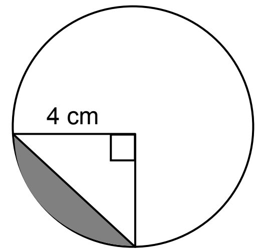
Measuring Composite Figures (pp. 3 of 3)

4. Find the perimeter and area of the shaded region of the composite figure. Round to the nearest hundredth when necessary.

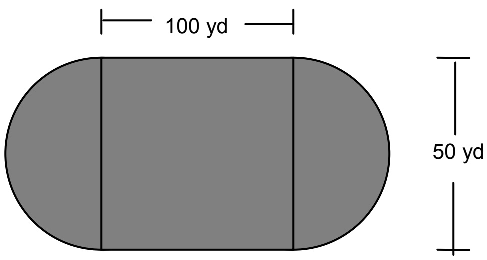
a.



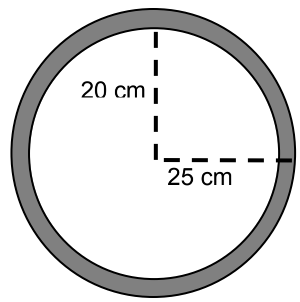
b.



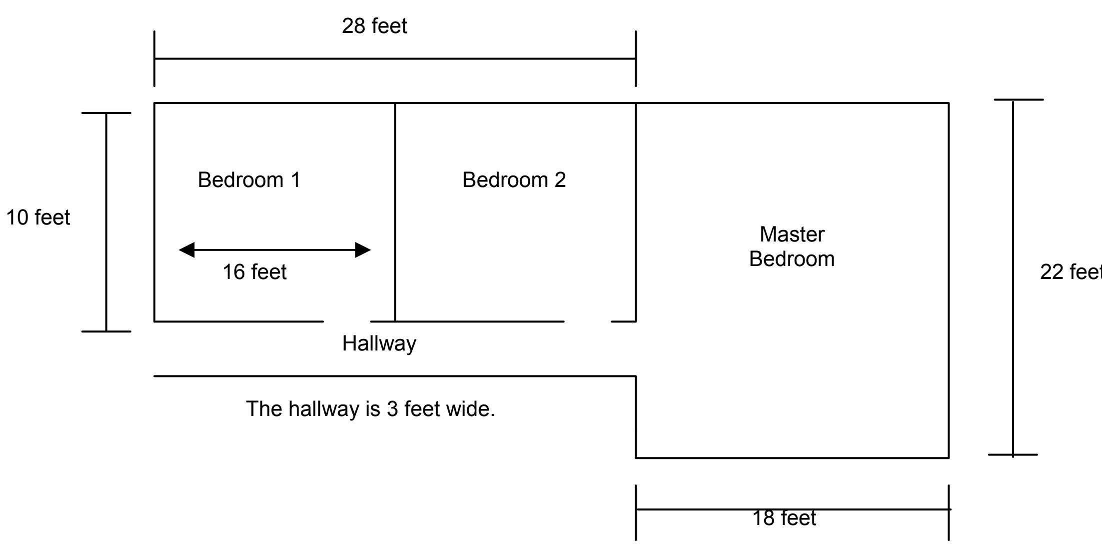
c.



d.



5. Jill is planning on carpeting the hall and bedrooms in her house. The floor plan is below. Find the amount of carpet she will need in square yards. Find the amount in square meters. (1.09 yards = 1 meter)



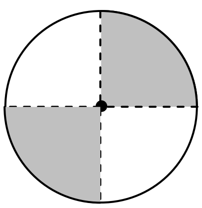
Area Models and Probability (pp. 1 of 3) KEY

Area models can be used to represent simple probabilities.

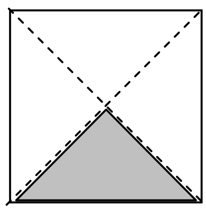
- The whole figure represents the total number of possible outcomes.
- The shaded part represents the desired outcomes.

Either circles or polygons can be used to create area models for probability. Figures must be divided into equal sections. Sections are shaded to represent the desired probability.

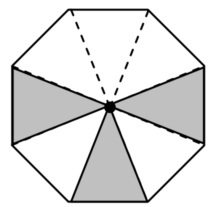
1. What simple probability is represented by each of the following area models? Assume divided sections within a figure are equal in area.



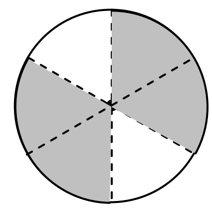
$\frac{2}{4}=\frac{1}{2}$



$\frac{1}{4}$

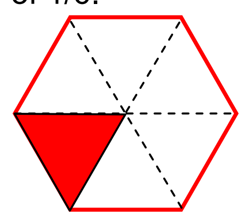


$\frac{3}{8}$

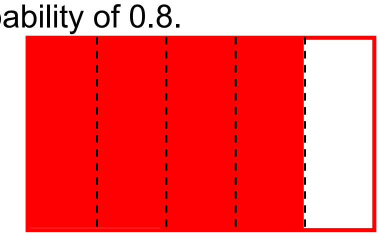


$\frac{4}{6}=\frac{2}{3}$

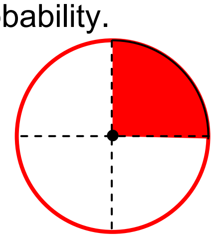
2. Use a hexagon split in equal sections to represent a probability of 1/6.
   Sample answer



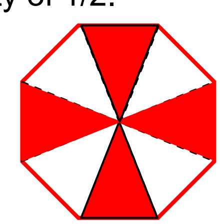
3. Use a rectangle split in equal sections to represent a probability of 0.8.
   Sample answer
   0.8 = 8/10 = 4/5



4. Use a circle split in equal sections to represent 25% probability.
   Sample answer



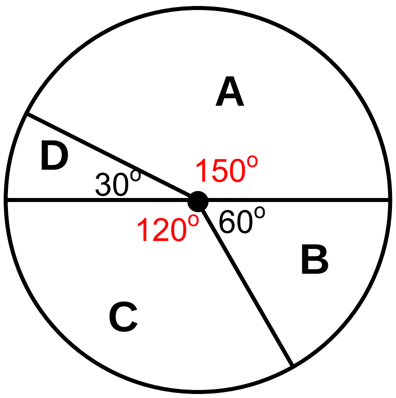
5. Use an octagon split in equal sections to represent a probability of 1/2.
   Sample answer



Area Models for Simple Probability (pp. 2 of 3) KEY

In the following problems calculate the necessary areas to determine probability.

6. After Gerald finishes his homework, he must play a game of chance to determine how he will spend the rest of the evening using the spinner below. Determine the probability and percent of his spending the evening in each event.
   Sector A – Playing with younger sibling
   Sector B – Going shopping at the mall
   Sector C – Cleaning the garage and workroom all evening
   Sector D – Playing in a band in a friend's garage



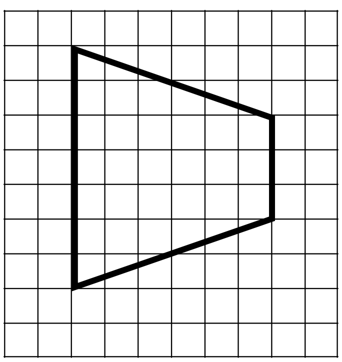
P(Playing with younger sibling) = $\frac{5}{12}, 41\frac{2}{3}\%$

P(Going shopping at mall) = $\frac{1}{6}, 16\frac{2}{3}\%$

P(Cleaning garage and workroom) = $\frac{1}{3}, 33\frac{1}{3}\%$

P(Playing in a band with friends) = $\frac{1}{12}, 8\frac{1}{3}\%$

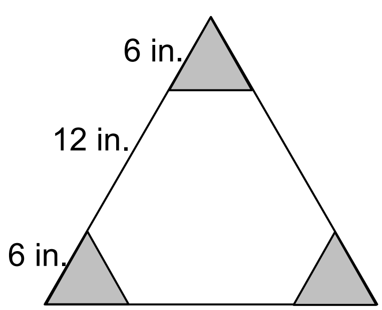
7. What is the probability that a randomly dropped counter would fall inside the trapezoid?



$A_{rect} = 10 \bullet 10 = 100$ sq. un.
$A_{trap} = (\tfrac{1}{2})h(b_1 + b_2)$
$A_{trap} = (\tfrac{1}{2})(6)(7 + 3) = 30$ sq. un.

P(Falling inside the trapezoid) = 30/100
P(Falling inside the trapezoid) = 3/10 or 30%

8. At a school fair students were challenged to hit one of the small, congruent, equilateral triangular regions on the large equilateral triangular board below with a dart. Find the probability of hitting a small triangle, if the dart hits the large triangular region.



P(hitting a shaded triangle) = 3/16 or 18.75%

Area Models for Simple Probability (pp. 3 of 3) KEY

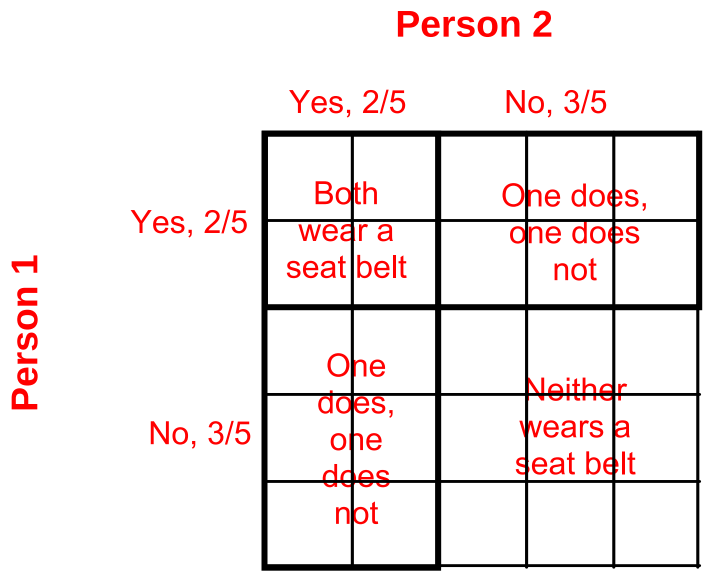
Area models can also be used to solve probability problems.

9. In the United States approximately $\frac{2}{5}$ of the people wear a seat belt while driving. If two people are chosen at random, what are the following probabilities?

   a. Both of them wear a seat belt? 4/25, 16%

   b. Only one of them wears a seat belt? 12/25, 48%

   c. Neither of them wears a seat belt? 9/25, 36%



10. The probability of passing a physics class is $\frac{3}{4}$. If Tom and Alice both take physics, what are the following probabilities? Use an area model to determine the probability.

    a. Both of them will pass? 9/16, 56.25%

    b. Only one of them will pass? 3/8, 37.5%

    c. Neither of them will pass? 1/16, 6.25%

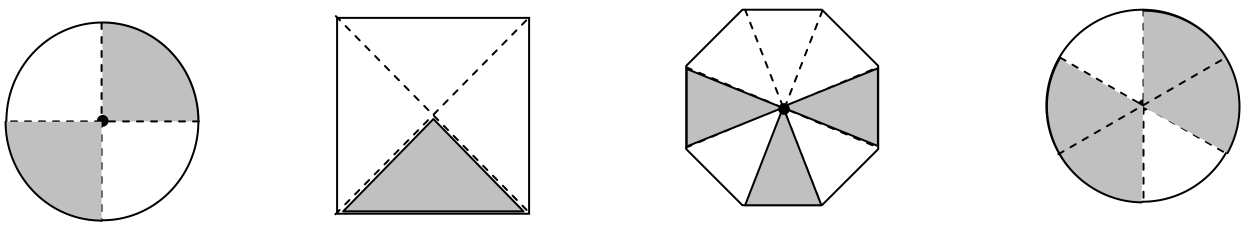
Area Models and Probability (pp. 1 of 3)

Area models can be used to represent simple probabilities.

- The whole figure represents the total number of possible outcomes.
- The shaded part represents the desired outcomes.

Either circles or polygons can be used to create area models for probability. Figures must be divided into equal sections. Sections are shaded to represent the desired probability.

1. What simple probability is represented by each of the following area models? Assume divided sections within a figure are equal in area.



____________ ____________ ____________ ____________

2. Use a hexagon split in equal sections to represent a probability of 1/6.

3. Use a rectangle split in equal sections to represent a probability of 0.8.

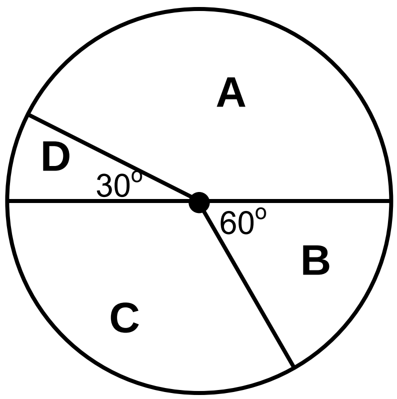
4. Use a circle split in equal sections to represent 25% probability.

5. Use an octagon split in equal sections to represent a probability of 1/2.

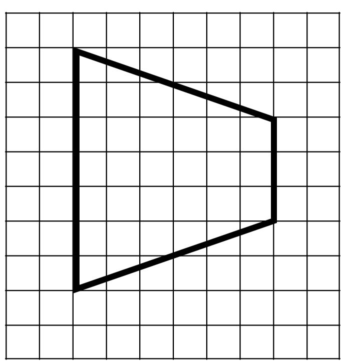
Area Models for Simple Probability (pp. 2 of 3)

In the following problems calculate the necessary areas to determine probability.

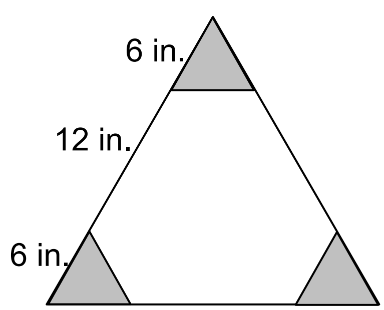
6. After Gerald finishes his homework, he must play a game of chance to determine how he will spend the rest of the evening using the spinner below. Determine the probability and percent of his spending the evening in each event.
   Sector A – Playing with younger sibling
   Sector B – Going shopping at the mall
   Sector C – Cleaning the garage and workroom all evening
   Sector D – Playing in a band in a friend's garage



7. What is the probability that a randomly dropped counter would fall inside the trapezoid?



8. At a school fair students were challenged to hit one of the small, congruent, equilateral triangular regions on the large equilateral triangular board below with a dart. Find the probability of hitting a small triangle, if the dart hits the large triangular region.



Area Models for Simple Probability (pp. 3 of 3)

Area models can also be used to solve probability problems.

9. In the United States approximately $\frac{2}{5}$ of the people wear a seat belt while driving. If two people are chosen at random, what are the following probabilities?

   a. Both of them wear a seat belt?

   b. Only one of them wears a seat belt?

   c. Neither of them wears a seat belt?

10. The probability of passing a physics class is $\frac{3}{4}$. If Tom and Alice both take physics, what are the following probabilities? Use an area model to determine the probability.

    a. Both of them will pass?

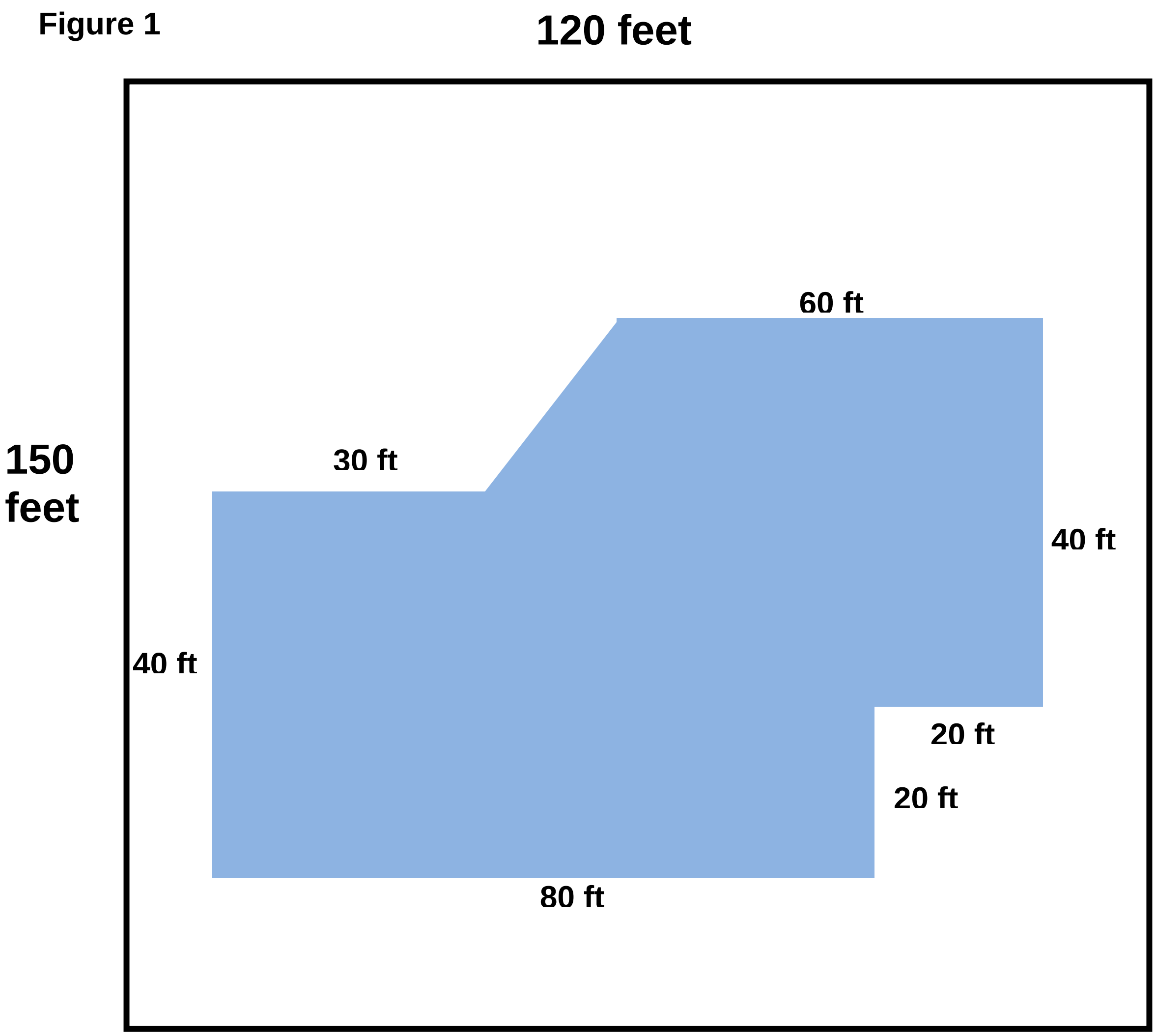
    b. Only one of them will pass?

    c. Neither of them will pass?

Poolside (pp. 1 of 2) KEY

Innovative Pools Inc. has been hired to build a pool in a rectangular backyard that is 120 feet by 150 feet. The rest of the area will be filled in with concrete stone. Assume angles that appear to be right angles are indeed right angles.

Figure 1



1. Find the surface area of the pool and its perimeter based on **Figure 1** above.

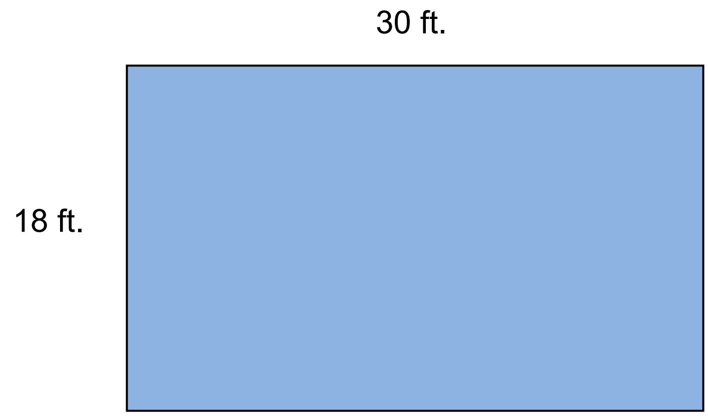
   P = 312.4 ft (approximately)

   A = 4900 ft^2

Poolside (pp. 2 of 2) KEY

Innovative Pools Inc. has been hired to build a rectangular pool for another customer. A diagram of the rectangular pool is below in **Figure 2**.

Figure 2



2. Find the surface area and perimeter of the pool based on **Figure 2** above.
   A = 540 ft^2 P = 96 ft

3. Suppose the customer tells the manager of Innovative Pools Inc. that he would like to double the size of the pool by doubling the dimensions of the pool. The manager of Innovative Pools tells the customer that doubling the dimensions of the pool will cost him at least four times as much money in materials and will more than double the surface area of the pool. The customer and the manager for Innovative Pools Inc. engage in a debate, each trying to convince the other of his point of view. Based on your knowledge of geometry, which of the two (the customer or the manager) has the correct point of view? Explain your reasoning.
   The manager has the correct point of view. The initial area is 540 ft^2.
   Doubling the dimensions makes the pool 36 ft by 60 f. resulting in an area of 2160 ft^2 which is four times the initial area.

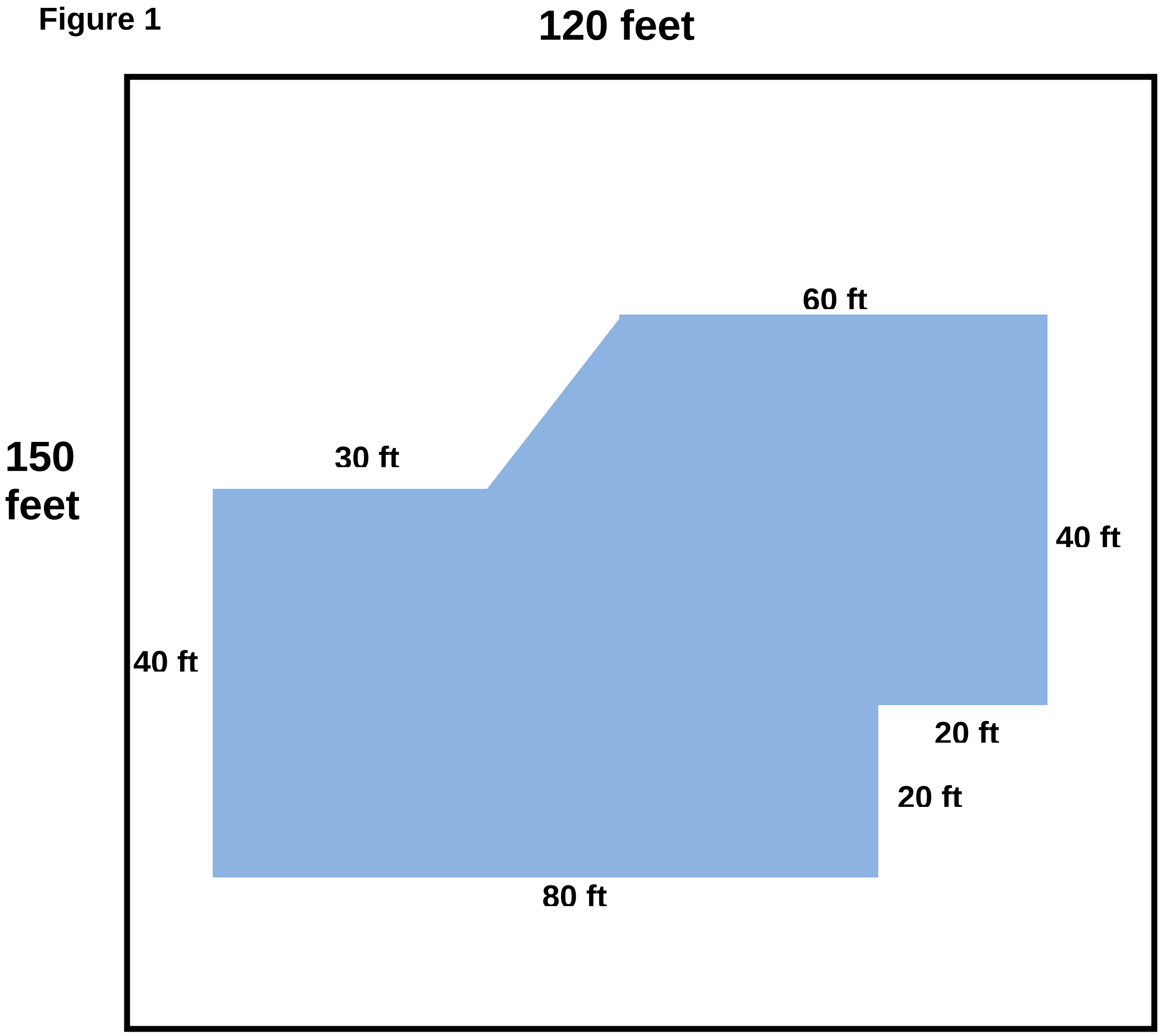
4. Using the initial dimensions from **Figure 2**, write a function to find the area of the rectangular pool as both the length and width are increased by the same amount *x*.
   $A = (18 + x)(30 + x)$ or $A = 540 + 48x + x^2$

5. To the nearest tenth of a foot, find the amount of increase needed to both the length and the width dimensions to double the area of the rectangular pool.
   9.4 ft.

Poolside (pp. 1 of 2)

Innovative Pools Inc. has been hired to build a pool in a rectangular backyard that is 120 feet by 150 feet. The rest of the area will be filled in with concrete stone. Assume angles that appear to be right angles are indeed right angles.

Figure 1

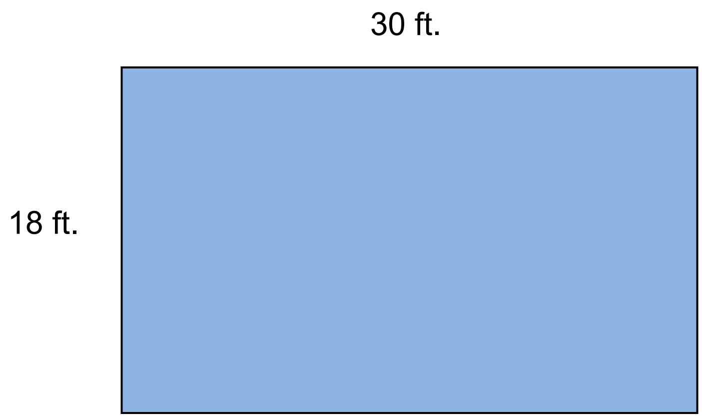


1. Find the surface area of the pool and its perimeter based on **Figure 1** above.

Poolside (pp. 2 of 2)

Innovative Pools Inc. has been hired to build a rectangular pool for another customer. A diagram of the rectangular pool is below in **Figure 2**.

Figure 2

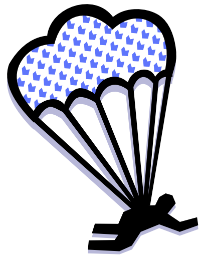


2. Find the surface area and perimeter of the pool based on **Figure 2** above.

3. Suppose the customer tells the manager of Innovative Pools Inc. that he would like to double the size of the pool by doubling the dimensions of the pool. The manager of Innovative Pools tells the customer that doubling the dimensions of the pool will cost him at least four times as much money in materials and will more than double the surface area of the pool. The customer and the manager for Innovative Pools Inc. engage in a debate, each trying to convince the other of his point of view. Based on your knowledge of geometry, which of the two (the customer or the manager) has the correct point of view? Explain your reasoning.

4. Using the initial dimensions from **Figure 2**, write a function to find the area of the rectangular pool as both the length and width are increased by the same amount *x*.

5. To the nearest tenth of a foot, find the amount of increase needed to both the length and the width dimensions to double the area of the rectangular pool.

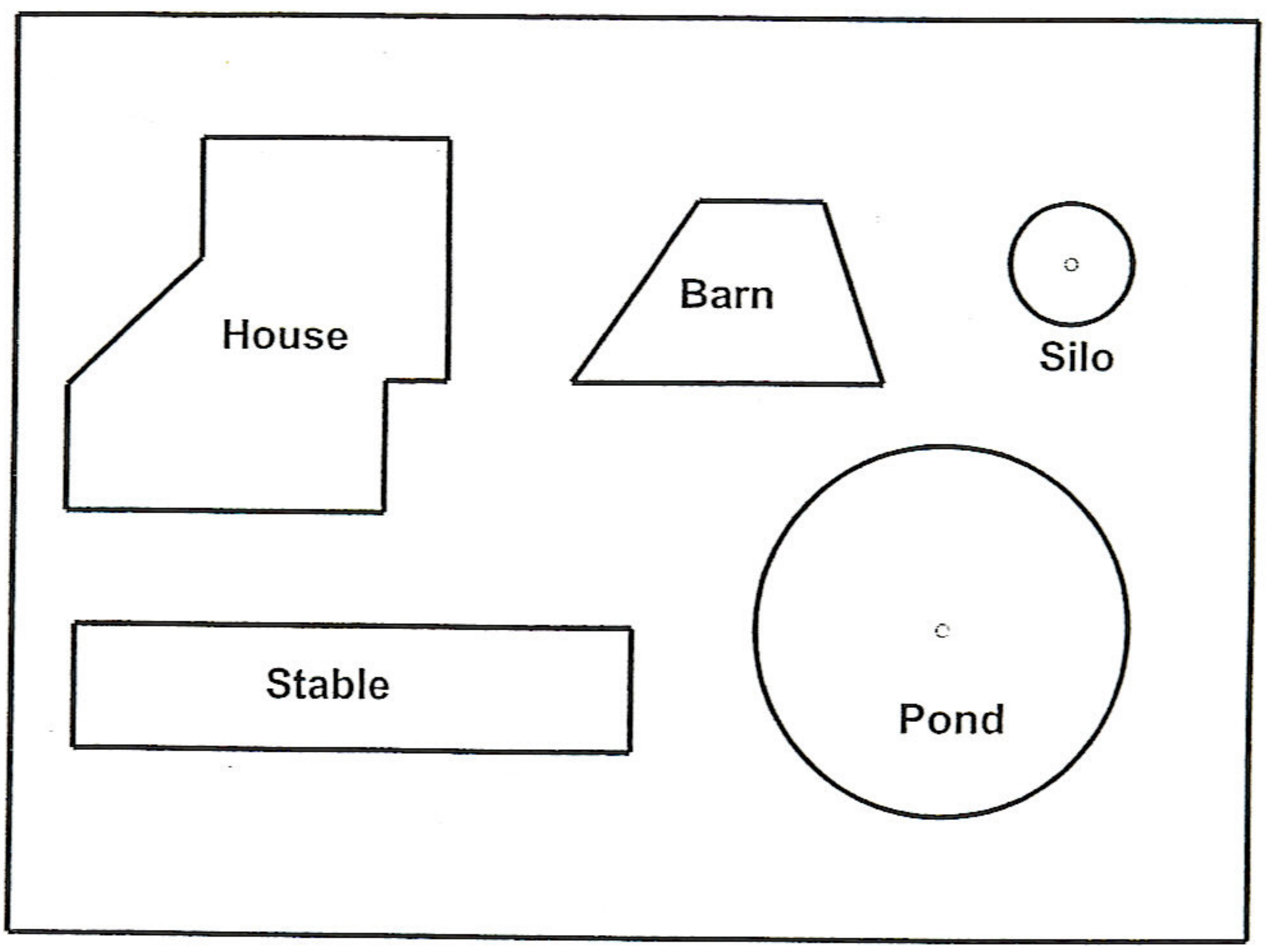


Sky Man

Sky Man is planning to perform his latest sky diving stunt over Old McDonald's farm. He wants to land on the open area shown and not on a building or in the water. Determine the probability that Sky Man will land in an open area. Assume right angles if they appear to be. Measure each necessary length in centimeters and convert to feet with the scale given below.

Scale: 1 centimeter = 20 feet

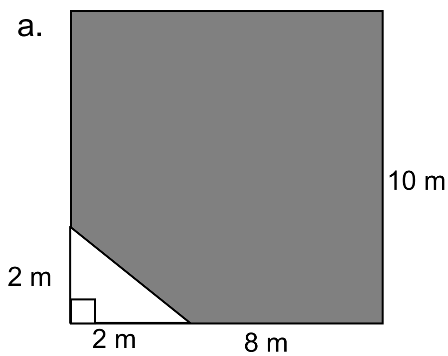
Total area = ___________________ square feet



Evaluating Composite Figures (pp. 1 of 4) KEY

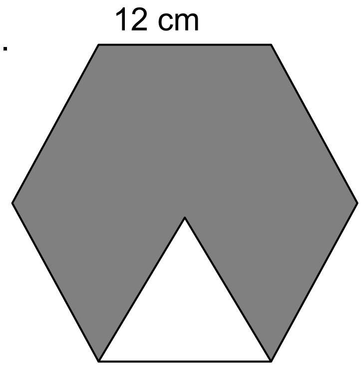
1. Find the perimeter and the area of the shaded region of each figure. Round answers to the nearest hundredth when necessary. Box answers.

a.



P = 38.83 m
A = 98 m^2

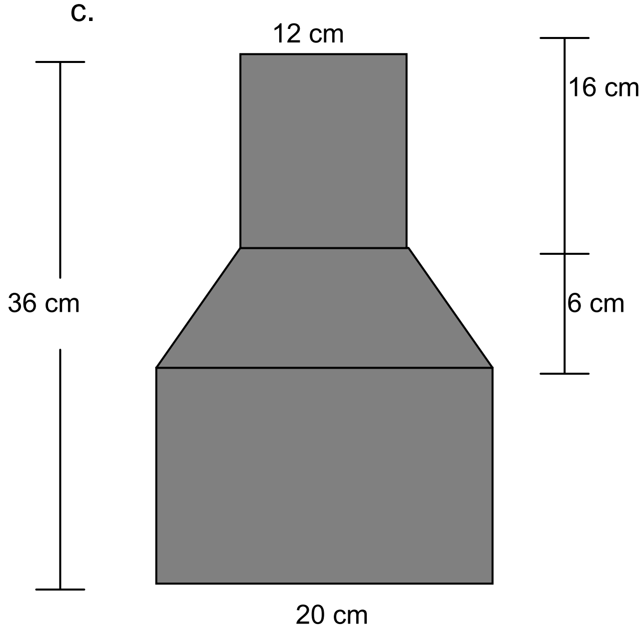
b.



Regular hexagon with inscribed equilateral triangle

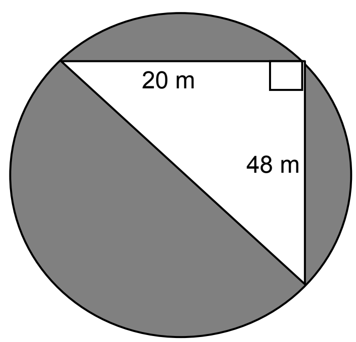
P = 84 cm
A = 311.77 cm^2

c.



P = 106.42 cm
A = 488 cm^2

d.

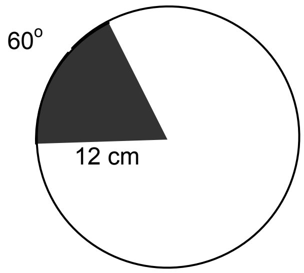


P = 283.36 m
A = 1643.72 m^2

Evaluating Composite Figures (pp. 2 of 4) KEY

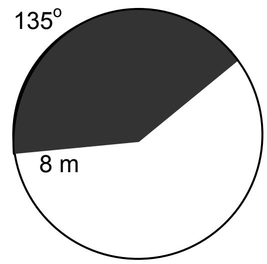
2. Find the arc length and the area of each shaded region or sector of the circle. Round to tenths.

a.



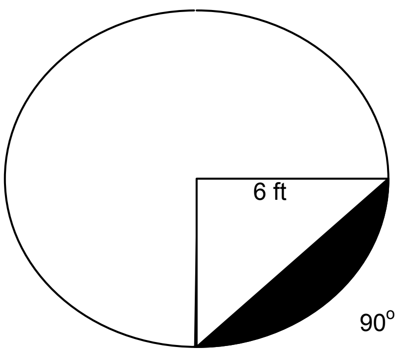
Arc Length = 12.57 cm
Area of shaded = 73.40 cm^2

b,



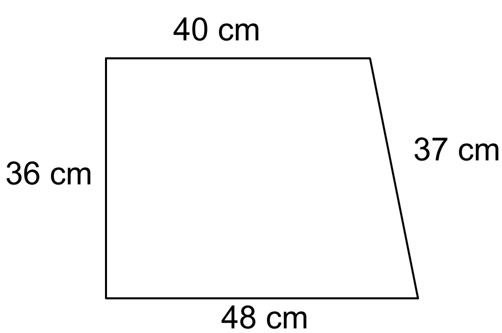
Arc Length = 18.85 m
Area of shaded = 75.40 m^2

c.



Arc Length = 9.42 cm
Area of shaded = 10.27 cm^2

3. The side window of an SUV is in the shape of the trapezoid shown below. A manufacture has been contracted to build 2,500 of the windows. What is the minimum number of square meters of glass he will need to complete the order? What is the minimum number of square feet he will need to complete the order? (3.28 feet = 1 meter)



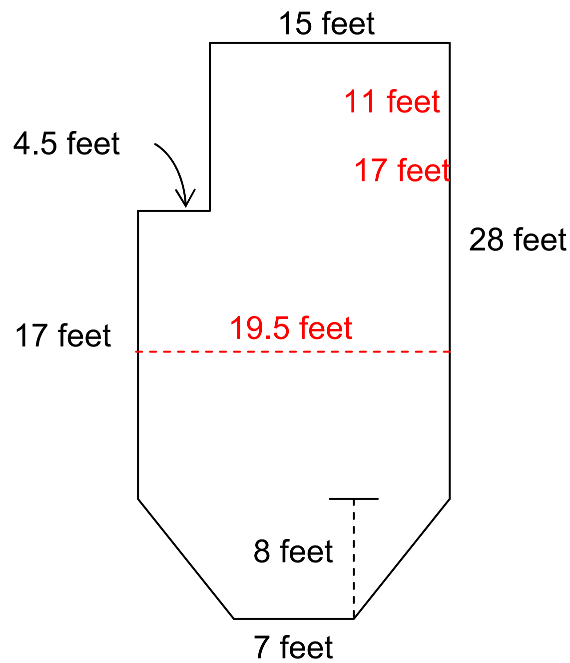
Area of window = (.5)(0.3)(0.40 + 0.48)
Area of window = 0.1584 square meters
0.1584(2500) = 396 square meter of glass

$396(3.28)^2$ = 4260.33 square feet of glass

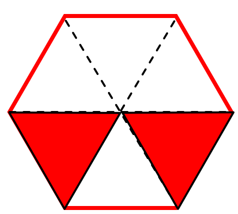
Evaluating Composite Figures (pp. 3 of 4) KEY

4. The Delaneys are going to carpet their family room. The diagram of the room is below. If the carpet they prefer costs $19.95 per **square yard**, approximately how much will it cost them to carpet the room?

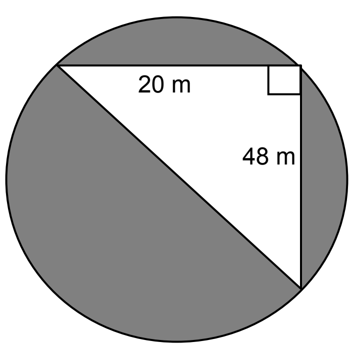
$A = 15 \bullet 11 + 17 \bullet 19.5 + (.5)(8)(19.5 + 7)$
$A = 602.5$ square feet
$A = \dfrac{602.5}{9}$ square yards
$A = 66.94$ square yards
$66.94(19.95) = \$1,355.54$



5. Use a hexagon split in equal sections to represent a probability of 1/3.
Sample:
1/3 = 2/6



6. If a dart must land in the circle, what is the probability of it landing in the triangular region?



Area of circle = 2123.72

Area of triangle = (0.5)(20)(48) = 480

P(landing in triangular region) = 480/2123.72

P(landing in triangular region) = .226 or 22.6%

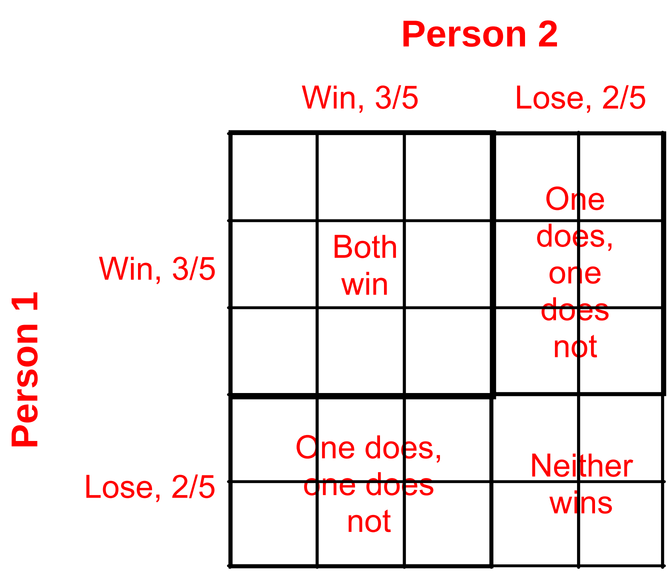
Evaluating Composite Figures (pp. 4 of 4)

7. The probability of winning a one person card game is $\frac{3}{5}$. If two people play the game, what are the following probabilities? Use an area model to determine the probability. Show all work.

   a. Both of them will win? 9/25 (36%)

   b. Only one of them will win and the other loses? 12/25 (48%)

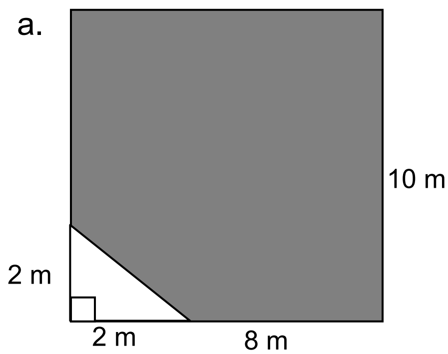
   c. Neither of them will win? 4/25 (16%)



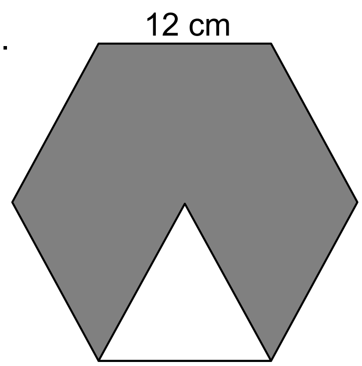
Evaluating Composite Figures (pp. 1 of 4)

1. Find the perimeter and the area of the shaded region of each figure. Round answers to the nearest hundredth when necessary. Box answers.

a.

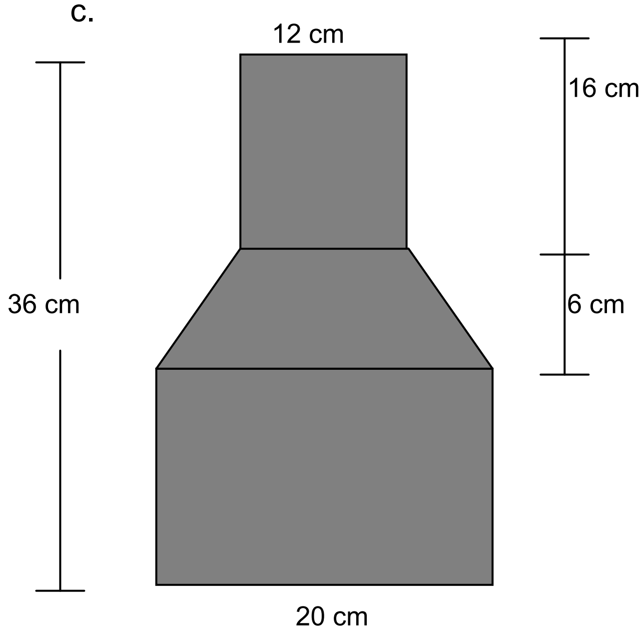


b.

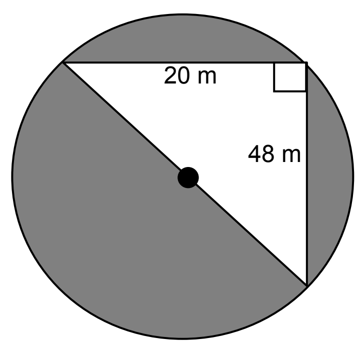


Regular hexagon with inscribed equilateral triangle

c.



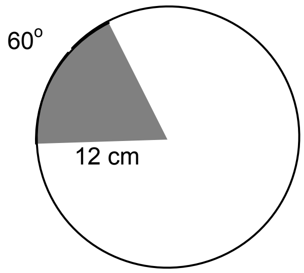
d.



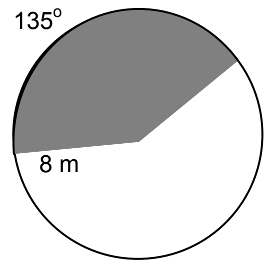
Evaluating Composite Figures (pp. 2 of 4)

2. Find the arc length and the area of each shaded region or sector of the circle. Round to tenths.

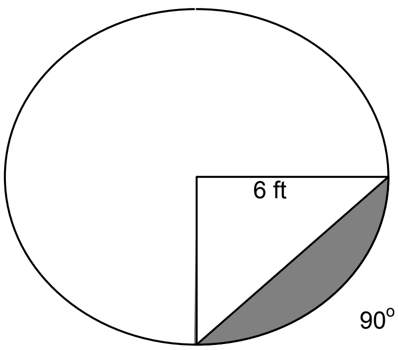
a.



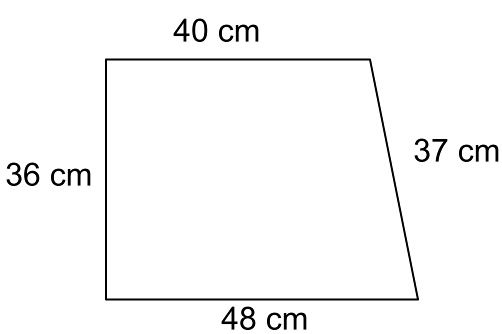
b,



c.

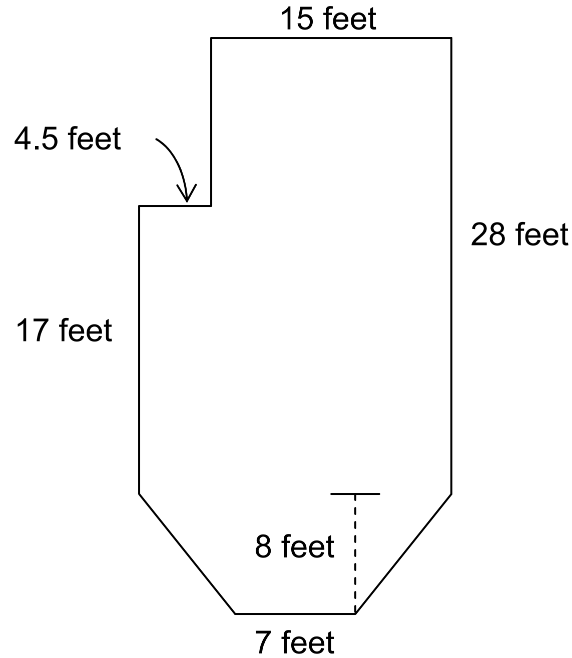


3. The side window of an SUV is in the shape of the trapezoid shown below. A manufacture has been contracted to build 2,500 of the windows. What is the minimum number of square meters of glass he will need to complete the order? What is the minimum number of square feet he will need to complete the order? (3.28 feet = 1 meter)



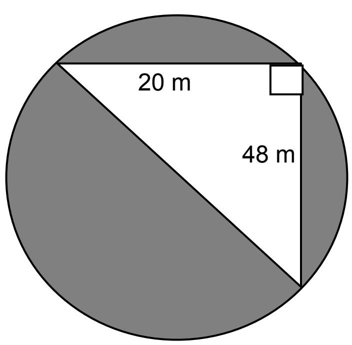
Evaluating Composite Figures (pp. 3 of 4)

4. The Delaneys are going to carpet their family room. The diagram of the room is below. If the carpet they prefer costs $19.95 per **square yard**, approximately how much will it cost them to carpet the room?



5. Use a hexagon split in equal sections to represent a probability of 1/3.

6. If a dart must land in the circle, what is the probability of landing in the triangular region?



Evaluating Composite Figures (pp. 4 of 4)

7. The probability of winning a one person card game is $\frac{3}{5}$. If two people play the game, what are the following probabilities? Use an area model to determine the probability. Show all work.

   a. Both of them will win?

   b. Only one of them will win and the other loses?

   c. Neither of them will win?

